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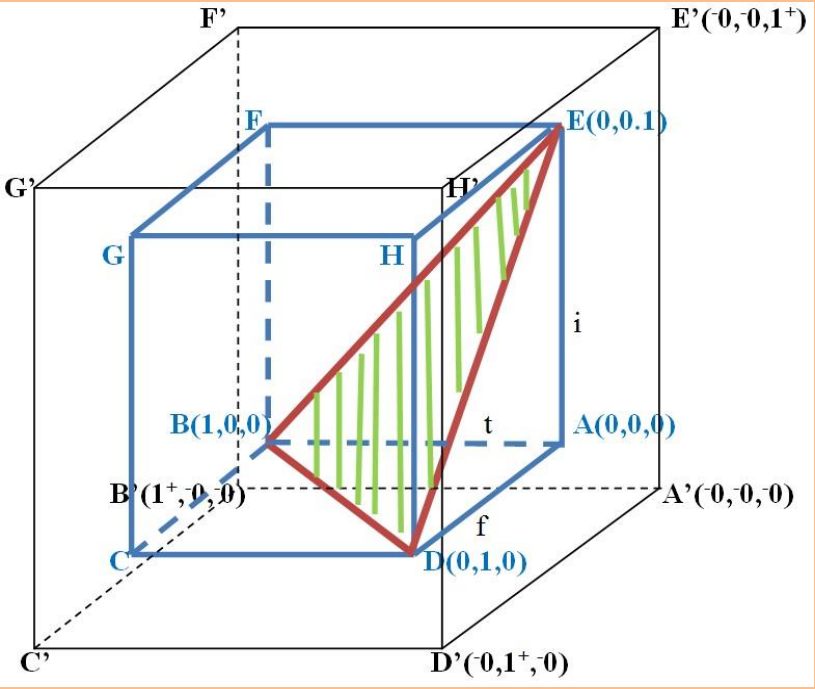

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The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (*T*), a degree of indeter

minacy (*I*), and a degree of falsity (*F*), where *T, I, F* are standard or non-standard subsets of $]^{-}0, 1^{+}[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Neutrosophic Correlation and Simple Linear Regression

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Abstract. Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosophic set, introduced by Smarandache in [7, 8]. Recently**,** Salama et al. **i**n [14, 15, 16, 32], introduced the concept of correlation coefficient of neutrosophic data. In this paper, we introduce and study the concepts of correlation and correlation coefficient of neutrosophic data in probability spaces and study some of their properties. Also, we introduce and study the neutrosophic simple linear regression model. Possible applications to data processing are touched upon.

Keywords: Correlation Coefficient, Fuzzy Sets, Neutrosophic Sets, Intuitionistic Fuzzy Sets, Neutrosophic Data; Neutrosophic Simple Linear Regression

1 Introduction

In 1965 [13], Zadeh first introduced the concept of fuzzy sets. Fuzzy set is very much useful and in this one real value $\mu_A(x) \in [0,1]$ is used to represent the grade of membership of a fuzzy set *A* defined on the crisp set *X*. After two decades Atanassov [18, 19, 20] introduced another type of fuzzy sets that is called intuitionistic fuzzy set (IFS) which is more practical in real life situations. Intuitionistic fuzzy sets handle incomplete information i.e., the grade of membership function and non-membership function but not the indeterminate information and inconsistent information which exists obviously in belief system. Smarandache [7,8] introduced another concept of imprecise data called neutrosophic sets. Salama et al. [1] introduced and studied the operations on neutrosophic sets and developed neutrosophic sets theory in [25, 26, 27, 28, 29, 30, 31, 32]. In statistical analysis, the correlation coefficient plays an important role in measuring the strength of the linear relationship between two variables. As the correlation coefficients defined on crisp sets have been much discussed, it is also very common in the theory of fuzzy sets to find the correlation between fuzzy sets, which accounts for the relationship between the fuzzy sets. Salama et al. [15] introduced the concepts of correlation and correlation coefficient of neutrosophic in the case of finite spaces. In this paper we discuss and derived a formula for correlation coefficient, defined on the domain of neutrosophic sets in probability spaces.

2 Terminologies

Definition 2.1 [13]

Let *X* be a fixed set. A fuzzy set *A* of *X* is an object having the form $A = \{(x, \mu_A(x)), x \in X\}$ where the function $\mu_A : X \to [0,1]$ define the degree of membership of the element $x \in X$ to the set *A*, Let *X* be a fixed set. An intuitionistic fuzzy set *A* of *X* is an object having the form: $A = \{(x, \mu_A(x), \nu_A(x)), x \in X\}$,

Where the function: $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define respectively the degree of membership and degree of non-membership of the element $x \in X$ to the set *A*, which is a subset of *X* and for every $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Let *X* be a non-empty fixed set. A neutrosophic set (NS) *A* is an object having the form:

$$A = \{(x, \mu_A(x), \gamma_A(x), \nu_A(x)), x \in X\}$$

where $\mu_A(x), \gamma_A(x)$ and $\nu_A(x)$ represent the degree of membership function, the degree of indeterminacy, and the degree of non membership function respectively of each element $x \in X$ to the set *A*.

In 1991, Gerstenkorn and Manko [24] defined the correlation of intuitionistic fuzzy sets *A* and *B* in a finite set $X = \{x_1, x_2, \ldots, x_n\}$ as follows:

$$C_{GM}(A,B) = \sum_{i=1}^{n} (\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)) \qquad (2.1)$$

and the correlation coefficient of fuzzy numbers *A*,*B* was given by: $\rho_{GM} = \dfrac{C_{GM}(A,B)}{\sqrt{T(A) \cdot T(B)}}$ (2.2)

where $T(A)=\sum_{i=1}^{n}(\mu_A^2(x_i)+\nu_A^2(x_i))$. (2.3)

Yu [4] defined the correlation of A and B in the collection $F([a,b])$ of all fuzzy numbers whose supports are included in a closed interval $[a,b]$ as follows:

$$C_Y(A,B)=\frac{1}{b-a}\int_a^b \mu_A(x)\mu_B(x)+\nu_A(x)\nu_B(x)dx, \quad (2.4)$$

where $\mu_A(x)+\nu_A(x)=1$ and the correlation coefficient of fuzzy numbers A,B was defined by

$$\rho_Y=\frac{C_Y(A,B)}{\sqrt{C_Y(A,A)\times C_Y(B,B)}}. \quad (2.5)$$

In 1995, Hong and Hwang [[5]] defined the correlation of intuitionistic fuzzy sets A and B in a probability space (X, B,P) as follows:

$$C_{HH}(A,B)=\int_X(\mu_A\mu_B+\nu_A\nu_B)dP \quad (2.6)$$

and the correlation coefficient of intuitionistic fuzzy numbers A,B was given by

$$\rho_{HH}=\frac{C_{HH}(A,B)}{\sqrt{C_{HH}(A,A)\cdot C_{HH}(B,B)}}. \quad (2.7)$$

Salama et al. [14] introduce the concept of positively and negatively correlated and used the concept of centroid to define the correlation coefficient of generalized intuitionistic fuzzy sets which lies in the interval [-1, 1], and satisfy the condition

$$\mu_A(x)\wedge\nu_A(x)\le 0.5, \quad \forall x\in X.$$

and the correlation coefficient of generalized intuitionistic fuzzy sets A and B was given by:

$$\rho_{HS}=\frac{C_{HS}(A,B)}{\sqrt{C_{HS}(A,A)\cdot C_{HS}(B,B)}}, \quad (2.8)$$

where $C_{HS}=m(\mu_A)m(\mu_B)+m(\nu_A)m(\nu_B)$ (2.9)

$$m(\mu_A)=\frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx},$$

$$m(\nu_A)=\frac{\int x\nu_A(x)dx}{\int \nu_A(x)dx},$$

$$m(\mu_B)=\frac{\int x\mu_B(x)dx}{\int \mu_B(x)dx},$$

$$m(\nu_B)=\frac{\int x\nu_B(x)dx}{\int \nu_B(x)dx}$$

Salama et al. [15] defined the correlation of neutrosophic data in a finite set $X=\{x_1,x_2,\ldots,x_n\}$ as follows:

$$C_{HS}(A,B)=\sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i)+\gamma_A(x_i)\gamma_B(x_i)) \quad (2.10)$$

and the correlation coefficient of fuzzy numbers A,B was given by:

$$\rho_{HS}=\frac{C_{HS}(A,B)}{\sqrt{T(A)\cdot T(B)}} \quad (2.11)$$

where

$$T(A)=\sum_{i=1}^{n}(\mu_A^2(x_i)+\nu_A^2(x_i)+\gamma_A^2(x_i)). \quad (2.12)$$

$$T(B)=\sum_{i=1}^{n}(\mu_B^2(x_i)+\nu_B^2(x_i)+\gamma_B^2(x_i)). \quad (2.13)$$

Salama et al. [16] introduce the concept of positively and negatively correlated and used the concept of centroid to define the correlation coefficient of neutrosophic sets which lies in the interval [-1, 1] was given by:

$$\rho_{HS}=\frac{C_{HS}(A,B)}{\sqrt{C_{HS}(A,A)\cdot C_{HS}(B,B)}} \quad (2.14)$$

where

$$C_{HS}=m(\mu_A)m(\mu_B)+m(\nu_A)m(\nu_B)+m(\gamma_A)m(\gamma_B) \quad (2.15)$$

$$m(\mu_A)=\frac{\int x\,\mu_A(x)dx}{\int \mu_A(x)dx}$$

$$m(\mu_B)=\frac{\int x\,\mu_B(x)dx}{\int \mu_B(x)dx}$$

$$m(\nu_A)=\frac{\int x\,\nu_A(x)dx}{\int \nu_A(x)dx} \qquad m(\nu_B)=\frac{\int x\,\nu_B(x)dx}{\int \nu_B(x)dx}$$

$$m(\gamma_A)=\frac{\int x\gamma_A(x)dx}{\int \gamma_A(x)dx} \qquad m(\gamma_B)=\frac{\int x\gamma_B(x)dx}{\int \gamma_B(x)dx}$$

3. Correlation Coefficient of Neutrosophic Sets

Let (X, B, P) be a probability space and A be a neutrosophic set in a probability space X,

$A=\{\langle x,\mu_A(x),\gamma_A(x),\nu_A(x)\rangle | x\in X\}$, where

$\mu_A(x),\gamma_A(x),\nu_A(x): X\to[0,1]$ are, respectively, Borel measurable functions satisfying

$^{-}0\le\mu_A(x)+\gamma_A(x)+\nu_A(x)\le 1^{+}$, where $]^{-}0,1^{+}[$ is non-standard unit interval [3].

Definition 3.1

For a neutrosophic sets A, B, we define the correlation of neutrosophic sets A and B as follows:

$$C(A,B)=\int_X(\mu_A\mu_B+\gamma_A\gamma_B+\nu_A\nu_B)dP\ .\ (3.1)$$

Where P is the probability measure over X. Furthermore, we define the correlation coefficient of neutrosophic sets A and B as follows:

$$\rho(A,B)=\frac{C(A,B)}{[T(A)\cdot T(B)]^{1/2}} \qquad (3.2)$$

where $T(A)=C(A,A)=\int_X(\mu_A^2+\gamma_A^2+\nu_A^2)dP$, and

$T(B)=C(B,B)=\int_X(\mu_B^2+\gamma_B^2+\nu_B^2)dP$. The following proposition is immediate from the definitions.

Proposition 3.1

For neutrosophic sets A and B in X, we have

i. $C(A,B)=C(B,A)$, $\rho(A,B)=\rho(B,A)$.

ii. If $A=B$, then $\rho(A,B)=1$.

The following theorem generalizes both Theorem 1 [24], Proposition 2.3 [[4]14] and Theorem 1[15] of which the proof is remarkably simple.

Theorem 3.1

For neutrosophic sets A and B in X, we have

$$0\le\rho(A,B)\le 1. \qquad (3.3)$$

Proof

The inequality $\rho(A,B)\ge 0$ is evident since $C(A,B)\ge 0$ and $T(A),T(B)\ge 0$. Thus, we need only to show that $\rho(A,B)\le 1$, or $C(A,B)\le[T(A)\cdot T(B)]^{1/2}$

For an arbitrary real number k, we have

$$0\le\int_x(\mu_A-k\mu_B)^2dP+\int_x(\gamma_A-k\gamma_B)^2dP+\int_x(\nu_A-k\nu_B)^2dP$$

$$=\int_X(\mu_A^2+\gamma_A^2+\nu_A^2)dP-2k\int_X(\mu_A\mu_B+\gamma_A\gamma_B+\nu_A\nu_B)dP$$

$$+k^2\int_X(\mu_B^2+\gamma_B^2+\nu_B^2)dP.$$

Thus, we can get:

$$\left(\int_X(\mu_A\mu_B+\gamma_A\gamma_B+\nu_A\nu_B)dP\right)^2\le\int_X(\mu_A^2+\gamma_A^2+\nu_A^2)dP\cdot\int_X(\mu_B^2+\gamma_B^2+\nu_B^2)dP$$

$$[C(A,B)]^2\le T(A)\cdot T(B),\ C(A,B)\le[T(A)\cdot T(B)]^{1/2}$$

Therefore, we have $\rho(A,B)\le 1$.

Theorem 3.2

$\rho(A,B)=1$ if and only if $A=cB$ for some $c\in IR$.

Proof

Considering the inequality in the proof of Theorem 3.1, then the equality holds if and only if

$$P\{\mu_A=c\mu_B\}=P\{\gamma_A=c\gamma_B\}=P\{\nu_A=c\nu_B\}=1$$

, for some $c\in IR$.

which completes the proof.

Theorem 3.3

$\rho(A,B)=0$ if and only if A and B are non-fuzzy sets and they satisfy the condition: $\mu_A+\mu_B=1$ or

$\gamma_A+\gamma_B=1$ or $\nu_A+\nu_B=1$.

Proof

Suppose that $\rho(A,B)=0$, then $C(A,B)=0$. Since $\mu_A\mu_B+\gamma_A\gamma_B+\nu_A\nu_B\ge 0$, then $C(A,B)=0$ implies $P(\mu_A\mu_B+\gamma_A\gamma_B+\nu_A\nu_B=0)=1$, which means that

$P(\mu_A\mu_B=0)=1$, $P(\gamma_A\gamma_B=0)=1$ and $P(\nu_A\nu_B=0)=1$.

If $\mu_A(x)=1$, then we can get $\mu_B(x)=0$ and $\gamma_A(x)=\nu_A(x)=0$. At the same time, if $\mu_B(x)=1$, then we can get $\mu_A(x)=0$ and $\gamma_B(x)=\nu_B(x)=0$, hence, we have $\mu_A+\mu_B=1$. Conversely, if A and B are non-fuzzy sets and $\mu_A+\mu_B=1$. If $\mu_A(x)=1$, then we can $\mu_B(x)=0$ and $\gamma_A(x)=\nu_A(x)=0$. On the other hand, if $\mu_B(x)=1$, then we can have $\mu_A(x)=0$ and $\gamma_B(x)=\nu_B(x)=0$, which implies $C(A,B)=0$.Similarly

we can give the proof when $\gamma_A + \gamma_B = 1$ or $\nu_A + \nu_B = 1$.

Theorem 3.4

If A is a non-fuzzy set, then $T(A) = 1$.

The proof is obvious.

Example

For a continuous universal set $X = [1,2]$, if two neutrosophic sets are written, respectively.

$A = \{(x, \mu_A(x), \nu_A(x), \gamma_A(x)) \mid x \in [1,2]\}$,

$B = \{(x, \mu_B(x), \nu_B(x), \gamma_A(x)) \mid x \in [1,2]\}$,

where

$$\mu_A(x) = 0.5(x-1), \quad 1 \le x \le 2,$$

$$\mu_B = 0.3(x-1), \quad 1 \le x \le 2$$

$$\nu_A(x) = 1.9 - 0.9x, \quad 1 \le x \le 2,$$

$$\nu_B = 1.4 - 0.4x, \quad 1 \le x \le 2$$

$$\gamma_A(x) = (5-x)/6, \quad 1 \le x \le 2,$$

$$\gamma_{\mathbf{B}}(x) = 0.5x - 0.3, \quad 1 \le x \le 2$$

Thus, we have $C(A,B) = 0.79556$, $T(A) = 0.79593$) and $T(B) = 0.9365.6$. Then we get $\rho(A,B) = 0.936506$.

It shows that neutrosophic sets A and B have a good positively correlated.

4. Neutrosophic linear regression model

Linear regression models are widely used today in business administration, economics, and engineering as well as in many other traditionally non-quantitative fields including social, health and biological sciences. Regression analysis is a methodology for analyzing phenomena in which a variable (output or response) depends on other variables called input (independent or explanatory) variables. Function is fitted to a set of given data to predict the value of dependent variable for a specified value of the independent variable. However, the phenomena in the real world cannot be analyzed exactly, because they depend on some uncertain factors and in some cases, it may be appropriate to use neutrosophic regression. Tanaka et al. (1982) [8] proposed the first linear regression analysis with a fuzzy model. According to this method, the regression coefficients are fuzzy numbers, which can be expressed as interval numbers with membership values. Since the regression coefficients are fuzzy numbers, the estimated dependent variable is also a fuzzy number. A collection of recent papers dealing with several approaches to fuzzy regression analysis can be found in Kacprzyk and Fedrizzi (1992)[17]]. Other contributions in this area are by Diamond (1988)[22], Tanaka and Ishibuchi (1991)[11], Savic and Pedrycz (1991)[16] and Ishibuchi (1992) [9]. Yen et al. (1999) [12] extended the results of a fuzzy linear regression model that uses symmetric triangular coefficient to one with non-symmetric fuzzy triangular coefficients.

In this section we will define the simple linear regression in neutrosophic set.

Definition 4.1

Assume that there is a random sample $(x_1, x_2, \ldots, x_n) \in X$, alone with the sequence of data; $((\mu_A(x_1), \gamma_A(x_1), \nu_A(x_1)), (\mu_B(x_1), \gamma_B(x_1), \nu_B(x_1)), \ldots, (\mu_A(x_n), (\mu_B(x_n), \gamma_B(x_n), \nu_B(x_n))$, as defined in Definition 2.3, $\mu_A(x), \gamma_A(x), \nu_A(x)$ represent the degree of membership function (namely $\mu_A(x)$), the degree of non-membership (namely $\nu_A(x)$), and the degree of indeterminacy (namely $\gamma_A(x)$) respectively of each element $x \in X$ to the set A. also $\mu_B(x), \gamma_B(x), \nu_B(x)$ represent the degree of membership function (namely $\mu_B(x)$), the degree of non-membership (namely $\nu_B(x)$), and the degree of indeterminacy (namely $\gamma_B(x)$) respectively of each element $x \in X$ to the set B.

Consider the following simple neutrosophic linear regression model: $A_i = \alpha B_i + \beta$,

where X_i denotes the independent variables, b the estimated neutrosophic intercept coefficient, a the estimated neutrosophic slope coefficients and Y_i the estimated neutrosophic output. As classical statistics linear regression we will define the neutrosophic coefficients a and b

$$\alpha = \frac{C(A,B)}{C(A,A)} \text{ and } \beta = E(\bar{Y}) - \alpha E(\bar{X}),$$

where $C(A, B)$ define in [15] as follows

$$C(A,B) = \sum_{i=1}^{n} (\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \gamma_A(x_i)\gamma_B(x_i)),$$

$$E(\bar{Y}) = \frac{1}{3}(\bar{\mu}_B(x_i) + \bar{\gamma}_B(x_i) + \bar{\nu}_B(x_i)), \text{and}$$

$$E(\bar{X}) = \frac{1}{3}(\bar{\mu}_A(x_i) + \bar{\gamma}_A(x_i) + \bar{\nu}_A(x_i)),$$

$$\bar{\mu}_A(x_i) = \frac{1}{n}\sum_{i=1}^{n} \mu_A(x_i), \quad \bar{\mu}_B(x_i) = \frac{1}{n}\sum_{i=1}^{n} \mu_B(x_i),$$

$$\bar{\gamma}_A(x_i) = \frac{1}{n}\sum_{i=1}^{n} \gamma_A(x_i), \quad \bar{\gamma}_B(x_i) = \frac{1}{n}\sum_{i=1}^{n} \gamma_B(x_i),$$

$$\bar{\nu}_A(x_i) = \frac{1}{n}\sum_{i=1}^{n} \nu_A(x_i), \quad \bar{\nu}_B(x_i) = \frac{1}{n}\sum_{i=1}^{n} \nu_B(x_i).$$

Example 4.1

In example [15], we comput that

$C(A, B) = 0.88$, $C(A, A) = T(A) = 0.83$,

$$\bar{\mu}_A(x_i) = \frac{1}{n}\sum_{i=1}^{n}\mu_A(x_i) = 0.4\,,$$

$$\bar{\mu}_B(x_i) = \frac{1}{n}\sum_{i=1}^{n}\mu_B(x_i) = 0.3\,,$$

$$\bar{\gamma}_A(x_i) = \frac{1}{n}\sum_{i=1}^{n}\gamma_A(x_i) = 0.3\,,$$

$$\bar{\gamma}_B(x_i) = \frac{1}{n}\sum_{i=1}^{n}\gamma_B(x_i) = 0.35\,,$$

$$\bar{\nu}_A(x_i) = \frac{1}{n}\sum_{i=1}^{n}\nu_A(x_i) = 0.35\,,$$

$$\bar{\nu}_B(x_i) = \frac{1}{n}\sum_{i=1}^{n}\nu_B(x_i) = 0.6\,.$$

$$E(\bar{Y}) = \frac{1}{3}(\bar{\mu}_B(x_i) + \bar{\gamma}_B(x_i) + \bar{\nu}_B(x_i)) = 0.42\,,$$

and $E(\bar{X}) = \frac{1}{3}(\bar{\mu}_A(x_i) + \bar{\gamma}_A(x_i) + \bar{\nu}_A(x_i)) = 0.35$,

then $\alpha = 1.06$ and $\beta = 0.49$.

Then neutrosophic linear regression model is given by

$$A_i = 1.06B_i + 0.49.$$

Conclusion

Our main goal of this work is propose a method to calculate the correlation coefficient of neutrosophic sets which lies in [0, 1], give us information for the degree of the relationship between the neutrosophic sets. Further, we discuss some of their properties and give example to illustrate our proposed method reasonable. Also we get the simple linear regression on neutrosophic sets.

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Generalization of Neutrosophic Rings and Neutrosophic Fields

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Abstract. In this paper we present the generalization of neutrosophic rings and neutrosophic fields. We also extend the neutrosophic ideal to neutrosophic biideal and neutrosophic N-ideal. We also find some new type of notions which are related to the strong or pure part of neutrosophy. We have given sufficient amount of examples to illustrate the theory of neutrosophic birings, neutrosophic N-rings with neutrosophic bifields and neutrosophic N-fields and display many properties of them in this paper.

Keywords: Neutrosophic ring, neutrosophic field, neutrosophic biring, neutrosophic N-ring, neutrosophic bifield neutrosophic N-field.

1 Introduction

Neutrosophy is a new branch of philosophy which studies the origin and features of neutralities in the nature. Florentin Smarandache in 1980 firstly introduced the concept of neutrosophic logic where each proposition in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic. In fact neutrosophic set is the generalization of classical sets, conventional fuzzy set $[1]$, intuitionistic fuzzy set $[2]$ and interval valued fuzzy set $[3]$. This mathematical tool is used to handle problems like imprecise, indeterminacy and inconsistent data etc. By utilizing neutrosophic theory, Vasantha Kandasamy and Florentin Smarandache dig out neutrosophic algebraic structures in $[11]$. Some of them are neutrosophic fields, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups, neutrosophic N-groups, neutrosophic semigroups, neutrosophic bisemigroups, neutrosophic N-semigroup, neutrosophic loops, neutrosophic biloops, neutrosophic N-loop, neutrosophic groupoids, and neutrosophic bigroupoids and so on.

In this paper we have tried to develop the the generalization of neutrosophic ring and neutrosophic field in a logical manner. Firstly, preliminaries and basic concepts are given for neutrosophic rings and neutrosophic fields. Then we presented the newly defined notions and results in neutrosophic birings and neutrosophic N-rings, neutrosophic bifields and neutosophic N-fields. Various types of neutrosophic biideals and neutrosophic N-ideal are defined and elaborated with the help of examples.

2 Fundamental Concepts

In this section, we give a brief description of neutrosophic rings and neutrosophic fields.

Definition: Let R be a ring. The neutrosophic ring $\langle R \cup I \rangle$ is also a ring generated by R and I under the operation of R, where I is called the neutrosophic element with property $I^2 = I$. For an integer n, $n + I$ and nI are neutrosophic elements and $0.I = 0$. I^{-1}, the inverse of I is not defined and hence does not exist.

Definition: Let $\langle R \cup I \rangle$ be a neutrosophic ring. A proper subset P of $\langle R \cup I \rangle$ is called a neutosophic subring if P itself a neutrosophic ring under the operation of $\langle R \cup I \rangle$.

Definition: Let T be a non-empty set with two binary operations $*$ and $\circ$. T is said to be a pseudo neutrosophic ring if

1. T contains element of the form $a + bI$ $(a, b$ are reals and $b \neq 0$ for atleast one value$)$.

2. $(T,*)$ is an abelian group.
3. $(T,\circ)$ is a semigroup.

Definition: Let $\langle R\cup I\rangle$ be a neutrosophic ring. A non-empty set P of $\langle R\cup I\rangle$ is called a neutrosophic ideal of $\langle R\cup I\rangle$ if the following conditions are satisfied.

1. P is a neutrosophic subring of $\langle R\cup I\rangle$, and
2. For every $p\in P$ and $r\in\langle R\cup I\rangle$, pr and $rp\in P$.

Definition: Let K be a field. The neutrosophic field generated by $\langle K\cup I\rangle$ which is denoted by $K(I)=\langle K\cup I\rangle$.

Definition: Let $K(I)$ be a neutrosophic field. A proper subset P of $K(I)$ is called a neutrosophic sufield if P itself a neutrosophic field.

3 **Neutrosophic Biring**

Definition **. Let $(BN(\mathrm{R}),*,\circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(\mathrm{R}),*,\circ)$ is said to be a neutrosophic biring if $BN(\mathrm{Rs})=R_1\cup R_2$ where atleast one of $(\mathrm{R}_1,*,\circ)$ or $(\mathrm{R}_2,*,\circ)$ is a neutrosophic ring and other is just a ring. R_1 and R_2 are proper subsets of $BN(\mathrm{R})$.

Example 2. Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{Z}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a neutrosophic ring under addition and multiplication. $(\mathrm{R}_2,*,\circ)$ is just a ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a neutrosophic biring.

Theorem: Every neutrosophic biring contains a corresponding biring.

Definition: Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ be a neutrosophic biring. Then $BN(\mathrm{R})$ is called a commutative neutrosophic biring if each $(\mathrm{R}_1,*,\circ)$ and $(\mathrm{R}_2,*,\circ)$ is a commutative neutrosophic ring.

Example 2. Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{Z}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a commutative neutrosophic ring and $(\mathrm{R}_2,*,\circ)$ is also a commutative ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a commutative neutrosophic biring.

Definition: Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ be a neutrosophic biring. Then $BN(\mathrm{R})$ is called a pseudo neutrosophic biring if each $(\mathrm{R}_1,*,\circ)$ and $(\mathrm{R}_2,*,\circ)$ is a pseudo neutrosophic ring.

Example 2. Let $BN(\mathrm{R})=(\mathrm{R}_1,+,\times)\cup(\mathrm{R}_2,+,\circ)$ where $(\mathrm{R}_1,+,\times)=\{0,I,2I,3I\}$ is a pseudo neutrosophic ring under addition and multiplication modulo 4 and $(\mathrm{R}_2,+,\times)=\{0,\pm1\mathrm{I},\pm2\mathrm{I},\pm3\mathrm{I},...\}$ is another pseudo neutrosophic ring. Thus $(BN(\mathrm{R}),+,\times)$ is a pseudo neutrosophic biring.

Theorem: Every pseudo neutrosophic biring is trivially a neutrosophic biring but the converse may not be true.

Definition 8. Let $(BN(\mathrm{R})=R_1\cup R_2;*,\circ)$ be a neutrosophic biring. A proper subset $(T,*,\circ)$ is said to be a neutrosophic subbiring of $BN(\mathrm{R})$ if

1) $T=T_1\cup T_2$ where $T_1=R_1\cap T$ and $T_2=R_2\cap T$ and
2) At least one of $(T_1,\circ)$ or $(T_2,*)$ is a neutrosophic ring.

Example: Let $BN(\mathrm{R})=(\mathrm{R}_1,*,\circ)\cup(\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ)=(\langle\mathbb{R}\cup I\rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ)=(\mathbb{C},+,\times)$. Let $P=P_1\cup P_2$ be a proper subset of $BN(R)$, where $P_1=(\mathbb{Q},+,\times)$ and $P_2=(\mathbb{R},+,\times)$. Clearly $(P,+,\times)$ is a neutrosophic subbiring of $BN(R)$.

Definition: If both $(\mathrm{R}_1,*)$ and $(\mathrm{R}_2,\circ)$ in the above definition ** are neutrosophic rings then we call

$(BN(\mathrm{R}),*,\circ)$ to be a strong neutrosophic biring.

Example 2. Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ) = (\langle \mathbb{Z} \cup I \rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ) = (\langle \mathbb{Q} \cup I \rangle,+,\times)$. Clearly R_1 and R_2 are neutrosophic rings under addition and multiplication. Thus $(BN(\mathrm{R}),*,\circ)$ is a strong neutrosophic biring.

Theorem. All strong neutrosophic birings are trivially neutrosophic birings but the converse is not true in general.

To see the converse, we take the following Example.

Example 2. Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ) = (\langle \mathbb{Z} \cup I \rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ) = (\mathbb{Q},+,\times)$. Clearly $(\mathrm{R}_1,*,\circ)$ is a neutrosophic ring under addition and multiplication. $(\mathrm{R}_2,*,\circ)$ is just a ring. Thus $(BN(\mathrm{R}),*,\circ)$ is a neutrosophic biring but not a strong neutrosophic biring.

Remark: A neutrosophic biring can have subbirings, neutrosophic subbirings, strong neutrosophic subbirings and pseudo neutrosohic subbirings.

Definition 8. Let $(BN(\mathrm{R}) = R_1 \cup R_2;*,\circ)$ be a neutrosophic biring and let $(T,*,\circ)$ is a neutrosophic subbiring of $BN(\mathrm{R})$. Then $(T,*,\circ)$ is called a neutrosophic biideal of $BN(R)$ if

1) $T = T_1 \cup T_2$ where $T_1 = R_1 \cap T$ and $T_2 = R_2 \cap T$ and
2) At least one of $(T_1,*,\circ)$ or $(T_2,*,\circ)$ is a neutrosophic ideal.

If both $(T_1,*,\circ)$ and $(T_2,*,\circ)$ in the above definition are neutrosophic ideals, then we call $(T,*,\circ)$ to be a strong neutrosophic biideal of $BN(R)$.

Example: Let $BN(\mathrm{R}) = (\mathrm{R}_1,*,\circ) \cup (\mathrm{R}_2,*,\circ)$ where $(\mathrm{R}_1,*,\circ) = (\langle \mathbb{Z}_{12} \cup I \rangle,+,\times)$ and $(\mathrm{R}_2,*,\circ) = (\mathbb{Z}_{16},+,\times)$. Let $P = P_1 \cup P_2$ be a neutrosophic subbiring of $BN(R)$, where $P_1 = \{0,6,2I,4I,6I,8I,10I,6+2I,...,6+10I\}$ and $P_2 = \{02I,4I,6I,8I,10I,12I,14I\}$. Clearly $(P,+,\times)$ is a neutrosophic biideal of $BN(R)$.

Theorem: Every neutrosophic biideal is trivially a neutrosophic subbiring but the converse may not be true.

Theorem: Every strong neutrosophic biideal is trivially a neutrosophic biideal but the converse may not be true.

Theorem: Every strong neutrosophic biideal is trivially a neutrosophic subbiring but the converse may not be true.

Theorem: Every strong neutrosophic biideal is trivially a strong neutrosophic subbiring but the converse may not be true.

Definition 8. Let $(BN(\mathrm{R}) = R_1 \cup R_2;*,\circ)$ be a neutrosophic biring and let $(T,*,\circ)$ is a neutrosophic subbiring of $BN(\mathrm{R})$. Then $(T,*,\circ)$ is called a pseudo neutrosophic biideal of $BN(R)$ if

1. $T = T_1 \cup T_2$ where $T_1 = R_1 \cap T$ and $T_2 = R_2 \cap T$ and
2. $(T_1,*,\circ)$ and $(T_2,*,\circ)$ are pseudo neutrosophic ideals.

Theorem: Every pseudo neutrosophic biideal is trivially a neutrosophic subbiring but the converse may not be true.

Theorem: Every pseudo neutrosophic biideal is trivially a strong neutrosophic subbiring but the converse may not be true.

Theorem: Every pseudo neutrosophic biideal is trivially a neutrosophic biideal but the converse may not be true.

Theorem: Every pseudo neutrosophic biideal is trivially a strong neutrosophic biideal but the converse may not be true.

4 Neutrosophic N -ring

Definition*. Let $\{\mathrm{N}(\mathrm{R}),*_1,...,*_2,\circ_1,\circ_2,...,\circ_N\}$ be a non-empty set with two N -binary operations defined on it. We call $N(R)$ a neutrosophic N -ring (N a positive integer) if the following conditions are satisfied.

1) $\mathrm{N}(\mathrm{R}) = R_1 \cup R_2 \cup ... \cup R_N$ where each R_i is a proper subset of $\mathrm{N}(\mathrm{R})$ i.e. $R_i \not\subset R_j$ or $R_j \not\subset R_i$ if $i \neq j$.

2) $(\mathrm{R}_i, *_i, \circ_i)$ is either a neutrosophic ring or a ring for $i = 1, 2, 3, ..., N$.

Example 2. Let $N(\mathrm{R}) = (\mathrm{R}_1, *, \circ) \cup (\mathrm{R}_2, *, \circ) \cup (\mathrm{R}_3, *, \circ)$ where $(\mathrm{R}_1, *, \circ) = (\langle \mathbb{Z} \cup I \rangle, +, \times)$, $(\mathrm{R}_2, *, \circ) = (\mathbb{Q}, +, \times)$ and $(R_3, *, \circ) = (Z_{12}, +, \times)$. Thus $(N(\mathrm{R}), *, \circ)$ is a neutrosophic N-ring.

Theorem: Every neutrosophic N-ring contains a corresponding N-ring.

Definition: Let $\mathrm{N}(\mathrm{R}) = \{\mathrm{R}_1 \cup R_2 \cup .... \cup \mathrm{R}_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. Then $N(\mathrm{R})$ is called a pseudo neutrosophic N-ring if each $(\mathrm{R}_i, *_i)$ is a pseudo neutrosophic ring where $i = 1, 2, ..., \mathrm{N}$.

Example 2. Let $N(\mathrm{R}) = (\mathrm{R}_1, +, \times) \cup (\mathrm{R}_2, +, \times) \cup (\mathrm{R}_3, +, \times)$ where $(\mathrm{R}_1, +, \times) = \{0, I, 2I, 3I\}$ is a pseudo neutrosophic ring under addition and multiplication modulo 4, $(\mathrm{R}_2, +, \times) = \{0, \pm 1\mathrm{I}, \pm 2\mathrm{I}, \pm 3\mathrm{I}, ...\}$ is a pseudo neutrosophic ring and $(\mathrm{R}_3, +, \times) = \{0, \pm 2\mathrm{I}, \pm 4\mathrm{I}, \pm 6\mathrm{I} ...\}$. Thus $(N(\mathrm{R}), +, \times)$ is a pseudo neutrosophic 3-ring.

Theorem: Every pseudo neutrosophic N-ring is trivially a neutrosophic N-ring but the converse may not be true.

Definition. If all the N-rings $(\mathrm{R}_i, *_i)$ in definition * are neutrosophic rings (i.e. for $i = 1, 2, 3, ..., N$) then we call $\mathrm{N}(\mathrm{R})$ to be a neutrosophic strong N-ring.

Example 2. Let $N(\mathrm{R}) = (\mathrm{R}_1, *, \circ) \cup (\mathrm{R}_2, *, \circ) \cup (\mathrm{R}_3, *, \circ)$ where $(\mathrm{R}_1, *, \circ) = (\langle \mathbb{Z} \cup I \rangle, +, \times)$, $(\mathrm{R}_2, *, \circ) = (\langle \mathbb{Q} \cup I \rangle, +, \times)$ and $(R_3, *, \circ) = (\langle \mathbb{Z}_{12} \cup I \rangle, +, \times)$. Thus $(N(\mathrm{R}), *, \circ)$ is a strong neutrosophic N-ring.

Theorem: All strong neutrosophic N-rings are neutrosophic N-rings but the converse may not be true.

Definition 13. Let $\mathrm{N}(\mathrm{R}) = \{\mathrm{R}_1 \cup R_2 \cup .... \cup \mathrm{R}_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $P = \{\mathrm{P}_1 \cup P_2 \cup .... \mathrm{P}_N, *_1, *_2, ..., *_N\}$ of $\mathrm{N}(\mathrm{R})$ is said to be a neutrosophic N-subring if $P_i = P \cap R_i, i = 1, 2, ..., N$ are subrings of R_i in which atleast some of the subrings are neutrosophic subrings.

Example: Let $N(\mathrm{R}) = (\mathrm{R}_1, *, \circ) \cup (\mathrm{R}_2, *, \circ) \cup (\mathrm{R}_3, *, \circ)$ where $(\mathrm{R}_1, *, \circ) = (\langle \mathbb{R} \cup I \rangle, +, \times)$, $(\mathrm{R}_2, *, \circ) = (\mathbb{C}, +, \times)$ and $(\mathrm{R}_2, *, \circ) = (\mathrm{Z}_{10}, +, \times)$ Let $P = P_1 \cup P_2 \cup P_3$ be a proper subset of $N(R)$, where $P_1 = (\mathbb{Q}, +, \times)$, $P_2 = (\mathbb{R}, +, \times)$ and $(\mathrm{R}_3, *, \circ) = \{0, 2, 4, 6, 8, \mathrm{I}, 2\mathrm{I}, 4\mathrm{I}, 6\mathrm{I}, 8\mathrm{I}\}$. Clearly $(P, +, \times)$ is a neutrosophic sub 3-ring of $N(R)$.

Definition 14. Let $\mathrm{N}(\mathrm{R}) = \{\mathrm{R}_1 \cup R_2 \cup .... \cup \mathrm{R}_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $T = \{\mathrm{T}_1 \cup T_2 \cup .... \cup \mathrm{T}_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ of $N(R)$ is said to be a neutrosophic strong sub N-ring if each $(T_i, *_i)$ is a neutrosophic subring of $(\mathrm{R}_i, *_i, \circ_i)$ for $i = 1, 2, ..., N$ where $T_i = R_i \cap T$.

Remark: A strong neutrosophic su N-ring is trivially a neutrosophic sub N-ring but the converse is not true.

Remark: A neutrosophic N-ring can have sub N-rings, neutrosophic sub N-rings, strong neutrosophic sub N-rings and pseudo neutrosohic sub N-rings.

Definition 16. Let $\mathrm{N}(\mathrm{R}) = \{\mathrm{R}_1 \cup R_2 \cup .... \cup \mathrm{R}_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $P = \{\mathrm{P}_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ where $P_t = P \cap R_t$ for $t = 1, 2, ..., N$ is said to be a neutrosophic N-ideal of $N(R)$ if the following conditions are satisfied.

1) Each it is a neutrosophic subring of $R_t, t = 1, 2, ..., N$.

2) Each it is a two sided ideal of R_t for $t = 1, 2, ..., N$.

If $(P_i, *_i, \circ_i)$ in the above definition are neutrosophic ideals, then we call $(P_i, *_i, \circ_i)$ to be a strong neutrosophic N-ideal of $N(R)$.

Theorem: Every neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

Theorem: Every strong neutrosophic N-ideal is trivially a neutrosophic N-ideal but the converse may not be true.

Theorem: Every strong neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

Theorem: Every strong neutrosophic biideal is trivially a strong neutrosophic subbiring but the converse may not be true.

Definition 16. Let $N(R) = \{R_1 \cup R_2 \cup .... \cup R_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N-ring. A proper subset $P = \{P_1 \cup P_2 \cup .... \cup P_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ where $P_t = P \cap R_t$ for $t = 1, 2, ..., N$ is said to be a pseudo neutrosophic N-ideal of $N(R)$ if the following conditions are satisfied.

1. Each it is a neutrosophic subring of $R_t, t = 1, 2, ..., N$.
2. Each $(P_i, *_i, \circ_i)$ is a pseudo neutrosophic ideal.

Theorem: Every pseudo neutrosophic N-ideal is trivially a neutrosophic sub N-ring but the converse may not be true.

Theorem: Every pseudo neutrosophic N-ideal is trivially a strong neutrosophic sub N-ring but the converse may not be true.

Theorem: Every pseudo neutrosophic N-ideal is trivially a neutrosophic N-ideal but the converse may not be true.

Theorem: Every pseudo neutrosophic N-ideal is trivially a strong neutrosophic N-ideal but the converse may not be true.

5 Neutrosophic Bi-Fields and Neutrosophic N-Fields

Definition **. Let $(BN(F), *, \circ)$ be a non-empty set with two binary operations $*$ and $\circ$. $(BN(F), *, \circ)$ is said to be a neutrosophic bifiel if $BN(F) = F_1 \cup F_2$ where atleast one of $(F_1, *, \circ)$ or $(F_2, *, \circ)$ is a neutrosophic field and other is just a field. F_1 and F_2 are proper subsets of $BN(F)$.

If in the above definition both $(F_1, *, \circ)$ and $(F_2, *, \circ)$ are neutrosophic fields, then we call $(BN(F), *, \circ)$ to be a neutrosophic strong bifield.

Example 2. Let $BN(F) = (F_1, *, \circ) \cup (F_2, *, \circ)$ where $(F_1, *, \circ) = (\langle \mathbb{C} \cup I \rangle, +, \times)$ and $(F_2, *, \circ) = (\mathbb{Q}, +, \times)$. Clearly $(F_1, *, \circ)$ is a neutrosophic field and $(F_2, *, \circ)$ is just a field. Thus $(BN(F), *, \circ)$ is a neutrosophic bifield.

Theorem: All strong neutrosophic bifields are trivially neutrosophic bifields but the converse is not true.

Definition 8. Let $BN(F) = (F_1 \cup F_2, *, \circ)$ be a neutrosophic bifield. A proper subset $(T, *, \circ)$ is said to be a neutrosophic subbifield of $BN(F)$ if

3) $T = T_1 \cup T_2$ where $T_1 = F_1 \cap T$ and $T_2 = F_2 \cap T$ and
4) At least one of $(T_1, \circ)$ or $(T_2, *)$ is a neutrosophic field and the other is just a field.

Example: Let $BN(F) = (F_1, *, \circ) \cup (F_2, *, \circ)$ where $(F_1, *, \circ) = (\langle \mathbb{R} \cup I \rangle, +, \times)$ and $(F_2, *, \circ) = (\mathbb{C}, +, \times)$. Let $P = P_1 \cup P_2$ be a proper subset of $BN(F)$, where $P_1 = (\mathbb{Q}, +, \times)$ and $P_2 = (\mathbb{R}, +, \times)$. Clearly $(P, +, \times)$ is a neutrosophic subbifield of $BN(F)$.

Definition*. Let $\{N(F), *_1, ..., *_2, \circ_1, \circ_2, ..., \circ_N\}$ be a non-empty set with two N-binary operations defined on it. We call $N(R)$ a neutrosophic N-field (N a positive integer) if the following conditions are satisfied.

1. $N(F) = F_1 \cup F_2 \cup ... \cup F_N$ where each F_i is a proper subset of $N(F)$ i.e. $R_i \not\subset R_j$ or $R_j \not\subset R_i$ if $i \neq j$.
2. $(R_i, *_i, \circ_i)$ is either a neutrosophic field or just a field for $i = 1, 2, 3, ..., N$.

If in the above definition each $(\mathrm{R}_i, *_i, \circ_i)$ is a neutrosophic field, then we call $N(R)$ to be a strong neutrosophic N-field.

Theorem: Every strong neutrosophic N-field is obviously a neutrosophic field but the converse is not true.

Definition 14. Let $\mathrm{N(F)} = \{\mathrm{F}_1 \cup F_2 \cup .... \cup F_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ be a neutrosophic N -field. A proper subset $T = \{\mathrm{T}_1 \cup T_2 \cup .... \cup \mathrm{T}_N, *_1, *_2, ..., *_N, \circ_1, \circ_2, ..., \circ_N\}$ of $N(\mathrm{F})$ is said to be a neutrosophic N -subfield if each $(T_i, *_i)$ is a neutrosophic subfield of $(\mathrm{F}_i, *_i, \circ_i)$ for $i = 1, 2, ..., N$ where $T_i = F_i \cap T$.

Conclusion

In this paper we extend neutrosophic ring and neutrosophic field to neutrosophic biring, neutrosophic N-ring and neutrosophic bifield and neutrosophic N-field. The neutrosophic ideal theory is extend to neutrosophic biideal and neutrosophic N-ideal. Some new type of neutrosophic ideals are discovered which is strongly neutrosophic or purely neutrosophic. Related examples are given to illustrate neutrosophic biring, neutrosophic N-ring, neutrosophic bifield and neutrosophic N-field and many theorems and properties are discussed.

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Cosine Similarity Measure of Interval Valued Neutrosophic Sets

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Abstract. In this paper, we define a new cosine similarity between two interval valued neutrosophic sets based on Bhattacharya's distance [19]. The notions of interval valued neutrosophic sets (IVNS, for short) will be used as vector representations in 3D-vector space. Based on the comparative analysis of the existing similarity measures for IVNS, we find that our proposed similarity measure is better and more robust. An illustrative example of the pattern recognition shows that the proposed method is simple and effective.

Keywords: Cosine Similarity Measure; Interval Valued Neutrosophic Sets

1. Introduction

The neutrsophic sets (**NS**), pioneered by F. Smarandache [1], has been studied and applied in different fields, including decision making problems [2, 3, 4 , 5, 23], databases [6-7], medical diagnosis problems [8] , topology [9], control theory [10], Image processing [11,12,13] and so on. The character of NSs is that the values of its membership function, non-membership function and indeterminacy function are subsets. The concept of neutrosophic sets generalizes the following concepts: the classic set, fuzzy set, interval valued fuzzy set, Intuitionistic fuzzy set, and interval valued intuitionistic fuzzy set and so on, from a philosophical point of view. Therefore, Wang et al [14] introduced an instance of neutrosophic sets known as single valued neutrosophic sets (SVNS), which were motivated from the practical point of view and that can be used in real scientific and engineering application, and provide the set theoretic operators and various properties of SVNSs. However, in many applications, due to lack of knowledge or data about the problem domains, the decision information may be provided with intervals, instead of real numbers. Thus, interval valued neutrosophic sets (IVNS), as a useful generation of NS, was introduced by Wang et al [15], which is characterized by a membership function, non-membership function and an indeterminacy function, whose values are intervals rather than real numbers. Also, the interval valued neutrosophic set can represent uncertain, imprecise, incomplete and inconsistent information which exist in the real world. As an important extension of NS, IVNS has many applications in real life [16, 17].

Many methods have been proposed for measuring the degree of similarity between neutrosophic set, S. Broumi and F. Smarandache [22] proposed several definitions of similarity measure between NS. P. Majumdar and S.K. Samanta [21] suggested some new methods for measuring the similarity between neutrosophic set. However, there is a little investigation on the similarity measure of IVNS, although some method on measure of similarity between intervals valued neutrosophic sets have been presented in [5] recently.

Pattern recognition has been one of the fastest growing areas during the last two decades because of its usefulness and fascination. In pattern recognition, on the basis of the knowledge of known pattern, our aim is to classify the unknown pattern. Because of the complex and uncertain nature of the problems. The problem pattern recognition is given in the form of interval valued neutrosophic sets.

In this paper, motivated by the cosine similarity measure based on Bhattacharya's distance [19], we propose a new method called "cosine similarity measure for interval valued neutrosophic sets. Also the proposed and existing similarity measures are compared to show that the proposed similarity measure is more reasonable than some similarity measures. The proposed similarity measure is applied to pattern recognition

This paper is organized as follow: In section 2 some basic definitions of neutrosophic set, single valued neutrosophic set, interval valued neutrosophic set and cosine similarity measure are presented briefly. In section 3, cosine similarity measure of interval valued neutrosophic sets and their proofs are introduced. In section 4, results of the proposed similarity measure and existing similarity measures are compared .In section 5, the proposed similarity measure is applied to deal with the problem related to medical diagnosis. Finally we conclude the paper.

2. Preliminaries

This section gives a brief overview of the concepts of neutrosophic set, single valued neutrosophic set, interval valued neutrosophic set and cosine similarity measure.

2.2 Neutrosophic Sets

Definition 2.1 [1]

Let U be an universe of discourse then the neutrosophic set A is an object having the form

A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, x ∈ U}, where the functions T, I, F : U→]−0, 1+[define respectively the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element x U to the set A with the condition.

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}. \qquad (1)$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. So instead of] −0, 1^{+}[we need to take the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NS, A_{NS} = {<x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > | x∈ X }

And B_{NS} = {<x, $T_B(x)$, $I_B(x)$, $F_B(x)$ > | x∈ X >the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ If and only if $T_A(x) \leq T_B(x)$, $I_A(x)$ $I_B(x)$, $F_A(x) \geq F_B(x)$ for any x ∈ X.

(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x)$ $= I_B(x)$, $F_A(x) = F_B(x)$ for any x ∈ X.

2.3.Single Valued Neutrosophic Sets

Definition 2.3 [14]

Let X be a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$, for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1].

When X is continuous, an SVNS A can be written as

$$A = \int_X \frac{< T_A(x), I_A(x), F_A(x) >}{x}, x \in X \qquad (2)$$

When X is discrete, an SVNS A can be written as

$$A = \sum_i^n \frac{< T_A(x_i), I_A(x_i), F_A(x_i) >}{x_i}, x_i \in X \qquad (3)$$

For two SVNS, A_{SVNS} = {<x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > | x ∈ X }

And B_{SVNS} ={<x, $T_A(x)$, $I_A(x)$, $F_A(x)$ > | x ∈ X }the two relations are defined as follows:

(1) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \leq T_B(x)$ $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$

(2) $A_{SVNS} = B_{SVNS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x)$ $= I_B(x)$, $F_A(x) = F_B(x)$ for any x ∈ X.

2.4 Interval Valued Neutrosophic Sets

Definition 2.4 [15]

Let X be a space of points (objects) with generic elements in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function $T_A(x)$, indeteminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x)$ ∈ [0, 1] .

For two IVNS, A_{INS} ={<x, [$T_A^L(x)$, $T_A^U(x)$] ,

[$I_A^L(x)$, $I_A^U(x)$],[$F_A^L(x)$, $F_A^U(x)$]> | x ∈ X }

And B_{INS} = {<x, ={<x, [$T_B^L(x)$, $T_B^U(x)$] ,

[$I_B^L(x)$, $I_B^U(x)$],[$F_B^L(x)$, $F_B^U(x)$]> | x ∈ X }>|x ∈ X } the two relations are defined as follows:

(1) $A_{INS} \subseteq B_{INS}$ if and only if $T_A^L(x) \leq T_B^L(x)$, $T_A^U(x) \leq T_B^U(x)$, $I_A^L(x) \geq I_B^L(x)$, $F_A^L(x) \geq F_B(x)$, $F_A^U(x) \geq F_B^U(x)$.

(2) $A_{INS} = B_{INS}$ if and only if , $T_A^L(x) = T_B^L(x)$,

$T_A^U(x) = T_B^U(x)$, $I_A^L(x) = I_B^L(x)$,

$I_A^U(x) = I_B^U(x)$, $F_A^L(x) = F_B^L(x)$, $F_A^U(x) = F_B^U(x)$ for any x ∈ X.

2.5 Cosine Similarity

Definition 2.5

Cosine similarity is a fundamental angle-based measure of similarity between two vectors of n dimensions using the cosine of the angle between them Candan and Sapino [20]. It measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them. Given two vectors of attributes, X= (x_1 , x_2 ,…, x_n) and Y= (y_1 , y_2 ,…, y_n), the cosine similarity, cosθ, is represented using a dot product and magnitude as

$$\mathbf{Cos\theta} = \frac{\sum_{i=}^{n} x_i y_i}{\sqrt{\sum_{i=}^{n} x_i^{\,2}} \sqrt{\sum_{i=}^{n} y_i^{\,2}}} \qquad (4)$$

In vector space, a cosine similarity measure based on Bhattacharya's distance [19] between two fuzzy set $\mu_A(x_i)$ and $\mu_B(x_i)$ defined as follows:

$$C_F(A,B) = \frac{\sum_{i=}^{n} \mu_A(x_i)\mu_B(x_i)}{\sqrt{\sum_{i=}^{n} \mu_A(x_i)^2}\sqrt{\sum_{i=}^{n} \mu_B(x_i)^2}} \qquad \textbf{(5)}$$

The cosine of the angle between the vectors is within the values between 0 and 1.

In 2-D vector space, J. Ye [18] defines cosine similarity measure between IFS as follows:

$$C_{IFS}(A,B) = \frac{\sum_{i=}^{n} \mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\sum_{i=}^{n} \mu_A(x_i)^2 + \nu_A(x_i)^2}\sqrt{\sum_{i=}^{n} \mu_B(x_i)^2 + \nu_B(x_i)^2}} \qquad \textbf{(6)}$$

III . Cosine Similarity Measure for Interval Valued Neutrosophic Sets.

The existing cosine similarity measure is defined as the inner product of these two vectors divided by the product of their lengths. The cosine similarity measure is the cosine of the angle between the vector representations of the two fuzzy sets. The cosine similarity measure is a classic measure used in information retrieval and is the most widely reported measures of vector similarity [19]. However, to the best of our Knowledge, the existing cosine similarity measures does not deal with interval valued neutrosophic sets. Therefore, to overcome this limitation in this section, a new cosine similarity measure between interval valued neutrosophic sets is proposed in 3-D vector space.

Let A be an interval valued neutrosophic sets in a universe of discourse X ={x}, the interval valued neutrosophic sets is characterized by the interval of membership $[T_A^L, T_A^U]$,the interval degree of non-membership $[F_A^L, F_A^U]$ and the interval degree of indeterminacy $[I_A^L, I_A^U]$ which can be considered as a vector representation with the three elements. Therefore, a cosine similarity measure for interval neutrosophic sets is proposed in an analogous manner to the cosine similarity measure proposed by J. Ye [18].

Definition 3.1 : Assume that there are two interval neutrosophic sets A and B in X $=\{x_1, x_2, ..., x_n\}$ Based on the extension measure for fuzzy sets, a cosine similarity measure between interval valued neutrosophic sets A and B is proposed as follows:

$$C_N(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)}{\sqrt{(\Delta T_A(x_i))^2+(\Delta I_A(x_i))^2+(\Delta F_A(x_i))^2}\sqrt{(\Delta T_B(x_i))^2+(\Delta I_B(x_i))^2+(\Delta F_B(x_i))^2}}. \qquad \textbf{(7)}$$

Where

$\Delta T_A(x_i) = T_A^L(x_i) + T_A^U(x_i)$, $\Delta T_B(x_i) = T_B^L(x_i) + T_B^U(x_i)$

$\Delta I_A(x_i) = I_A^L(x_i) + I_A^U(x_i)$, $\Delta I_B(x_i) = I_B^L(x_i) + I_B^U(x_i)$

And $\Delta F_A(x_i) = F_A^L(x_i) + F_A^U(x_i)$,

$\Delta F_B(x_i) = F_B^U(x_i) + F_B^U(x_i)$

Proposition 3.2

Let A and B be interval valued neutrosophic sets then

i. $\mathbf{0} \le C_N(A,B) \le \mathbf{1}$

ii. $C_N(A,B) = C_N(B,A)$

iii. $C_N(A,B) = 1$ if A= B i.e

$T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$

$I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and

$F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,…., n

Proof : (i) it is obvious that the proposition is true according to the cosine valued

(ii) it is obvious that the proposition is true.

(iii) when A =B, there are

$T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$

$I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and

$F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,…., n

, So there is $C_N(A,B) = 1$

If we consider the weights of each element x_i , a weighted cosine similarity measure between IVNSs A and B is given as follows:

$$C_{WN}(A,B) = \frac{1}{n}\sum_{i=1}^{n} w_i \frac{\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)}{\sqrt{(\Delta T_A(x_i))^2+(\Delta I_A(x_i))^2+(\Delta F_A(x_i))^2}\sqrt{(\Delta T_B(x_i))^2+(\Delta I_B(x_i))^2+(\Delta F_B(x_i))^2}}. \qquad \textbf{(8)}$$

Where $w_i \in [0,1]$,i =1,2,…,n ,and $\sum_{i=1}^{n} w_i = 1$.

If we take $w_i = \frac{1}{n}$, i =1,2,…,n , then there is $C_{WN}(A,B) = C_N(A,B)$.

The weighted cosine similarity measure between two IVNSs A and B also satisfies the following properties:

i. $\mathbf{0} \le C_{WN}(A,B) \le \mathbf{1}$

ii. $C_{WN}(A,B) = C_{WN}(B,A)$

iii. $C_{WN}(A,B) = 1$ if A= B i.e

$T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$

$I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and

$F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$ for i=1,2,…., n

Proposition 3.3

Let the distance measure of the angle as **d(A,B)= arcos** $C_N(A,B)$ **,**then it satisfies the following properties.

i. d(A, B) ≥ 0, if $\mathbf{0} \le C_N(A,B) \le \mathbf{1}$

ii. d(A, B) = **arcos(1) = 0, if** $C_N(A,B) = 1$

iii. d(A, B) = d(B, A) if $C_N(A,B) = C_N(B,A)$

iv. d(A, C) ≤ d(A, B) + d(B, C) if A ⊆ B ⊆ C for any interval valued neutrosophic sets C.

Proof : obviously, d(A,B) satisfies the (i) – (iii). In the following , d(A,B) will be proved to satisfy the (iv).

For any C = { x_i }, A ⊆ B ⊆ C since Eq (7) is the sum of terms. Let us consider the distance measure of the angle between vectors:

d_i (A(x_i), B(x_i)) = **arcos(** C_N (A(x_i), B(x_i))**,**

d_i (B(x_i), C(x_i)) = **arcos(** C_N (B(x_i), C(x_i))**, and**
d_i (A(x_i), C(x_i)) = **arcos(** C_N (A(x_i), C(x_i))**,,** for i=1, 2, .., n, where

$$C_N(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{\Delta T_A(x_i)\Delta T_B(x_i)+\Delta I_A(x_i)\Delta I_B(x_i)+\Delta F_A(x_i)\Delta F_B(x_i)}{\sqrt{(\Delta T_A(x_i))^2+(\Delta I_A(x_i))^2+(\Delta F_A(x_i))^2}\sqrt{(\Delta T_B(x_i))^2+(\Delta I_B(x_i))^2+(\Delta F_B(x_i))^2}}. \quad \textbf{(9)}$$

$$C_N(B,C)=\frac{1}{n}\sum_{i=1}^{n}\frac{\Delta T_B(x_i)\Delta T_C(x_i)+\Delta I_B(x_i)\Delta I_C(x_i)+\Delta F_B(x_i)\Delta F_C(x_i)}{\sqrt{(\Delta T_B(x_i))^2+(\Delta I_B(x_i))^2+(\Delta F_B(x_i))^2}\sqrt{(\Delta T_C(x_i))^2+(\Delta I_C(x_i))^2+(\Delta F_C(x_i))^2}}. \quad \textbf{(10)}$$

$$C_N(A,C)=\frac{1}{n}\sum_{i=1}^{n}\frac{\Delta T_A(x_i)\Delta T_C(x_i)+\Delta I_A(x_i)\Delta I_C(x_i)+\Delta F_A(x_i)\Delta F_C(x_i)}{\sqrt{(\Delta T_A(x_i))^2+(\Delta I_A(x_i))^2+(\Delta F_A(x_i))^2}\sqrt{(\Delta T_C(x_i))^2+(\Delta I_C(x_i))^2+(\Delta F_C(x_i))^2}}. \quad \textbf{(11)}$$

For three vectors

A(x_i)= < x_i , [$T_A^L(x_i)$, $T_A^U(x_i)$], [$I_A^L(x_i)$, $I_A^U(x_i)$], [$F_A^L(x_i)$, $F_A^U(x_i)$] >

B(x_i) = < [$T_B^L(x_i)$, $T_B^U(x_i)$], [$I_B^L(x_i)$, $I_b^U(x_i)$], [$F_B^L(x_i)$, $F_B^U(x_i)$] >

C(x_i) = < [$T_C^L(x_i)$, $T_C^U(x_i)$], [$I_C^L(x_i)$, $I_C^U(x_i)$], [$F_C^L(x_i)$, $F_C^U(x_i)$] > in a plane

If A(x_i) ⊆ B (x_i) ⊆ C (x_i) (i =1, 2,…, n), then it is obvious that d(A(x_i), C(x_i)) ≤ d(A(x_i), B(x_i)) + d(B(x_i), C(x_i)), According to the triangle inequality. Combining the inequality with E.q (7), we can obtain d(A, C) ≤ d(A, B) + d(B, C). Thus, d(A,B) satisfies the property (iv). So we have finished the proof.

IV. Comparison of New Similarity Measure with the Existing Measures.

Let A and B be two interval neutrosophic set in the universe of discourse X={ x_1 , x_2 ,…, x_n }. For the cosine similarity and the existing similarity measures of interval valued neutrosophic sets introduced in [5, 21], they are listed as follows:

Pinaki's similarity I [21]

$$S_{PI} = \frac{\sum_{i=1}^{n}\{\min\{T_A(x_i)\cdot T_B(x_i)\}+\min\{I_A(x_i)\cdot I_B(x_i)\}+\min\{F_A(x_i)\cdot F_B(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_A(x_i)\cdot T_B(x_i)\}+\max\{I_A(x_i)\cdot I_B(x_i)\}+\max\{F_A(x_i)\cdot F_B(x_i)\}\}}. \quad \textbf{(12)}$$

Also ,P. Majumdar [21] proposed weighted similarity measure for neutrosophic set as follows:

$$S_{PII}=\frac{\sum_{i=1}^{n}w_i(T_A(x_i)\cdot T_B(x_i)+I_A(x_i)\cdot I_B(x_i)+F_A(x_i)\cdot F_B(x_i))}{Max(w_i\sqrt{T_A(x_i)^2+T_A(x_i)^2+T_A(x_i)^2},w_i\sqrt{T_B(x_i)^2+I_B(x_i)^2+F_B(x_i)^2})} \quad \textbf{(13)}$$

Where, S_{PI} , S_{PII} denotes Pinaki's similarity I and Pinaki's similarity II

Ye's similarity [5] is defined as the following:

$$S_{ye}(A,B) = 1-\sum_{i=1}^{n}w_i\begin{bmatrix}|\inf T_A(x_i)-\inf T_B(x_i)|+|\sup T_A(x_i)-\sup T_B(x_i)|\\+|\inf I_A(x_i)-\inf i_B(x_i)|+|\sup I_A(x_i)-\sup I_B(x_i)|\\+|\inf F_A(x_i)-\inf F_B(x_i)|+|\sup F_A(x_i)-\sup F_B(x_i)|\end{bmatrix} \quad (14)$$

Example 1:

Let A = {<x, (0.2, 0.2 0.3)>} and B= {<x, (0.5, 0.2 0.5)>}

Pinaki similarity I = 0.58

Pinaki similarity II (with w_i =1) = 0.29

Ye similarity **(w**ith w_i =1) = 0.83

Cosine similarity $C_N(A,B)$ = **0.95**

Example 2:

Let A= {<x, ([0.2, 0.3], [0.5, 0.6] ,[0.3, 0.5])>} and B{<x, ([0.5, 0.6], [0.3, 0.6] ,[0.5, 0.6])>}

Pinaki similarty I = NA

Pinaki similarty II (with w_i =1) = NA

Ye similarity **(**with w_i =1) =0.81

Cosine similarity $C_N(A,B)$ = **0.92**

On the basis of computational study. J.Ye [5] have shown that their measure is more effective and reasonable .A similar kind of study with the help of the proposed new measure

based on the cosine similarity, has been done and it is found that the obtained results are more refined and accurate. It may be observed from the example 1 and 2 that the values of similarity measures are more closer to 1 with $C_N(A,B)$,the proposed similarity measure. This implies that we may be more deterministic for correct diagnosis and proper treatment.

V. Application of Cosine Similarity Measure for Interval Valued Neutrosophic Numbers to Pattern Recognition

In order to demonstrate the application of the proposed cosine similarity measure for interval valued neutrosophic numbers to pattern recognition, we discuss the medical diagnosis problem as follows:
For example the patient reported temperature claiming that the patient has temperature between 0.5 and 0.7 severity /certainty, some how it is between 0.2 and 0.4 indeterminable if temperature is cause or the effect of his current disease. And it between 0.1 and 0.2 sure that temperature has no relation with his main disease. This piece of information about one patient and one symptom may be written as:
(patient , Temperature) = <[0.5, 0.7], [0.2 ,0.4], [0.1, 0.2]>
(patient , Headache) = < [0.2, 0.3], [0.3 ,0.5], [0.3, 0.6]>
(patient , Cough) = <[0.4, 0.5], [0.6 ,0.7], [0.3, 0.4]>
Then, P = {< x_1 , [0.5, 0.7], [0.2 ,0.4], [0.1, 0.2] >, < x_2 , [0.2, 0.3], [0.3, 0.5], [0.3, 0.6] > ,< x_3 , [0.4, 0.5], [0.6 ,0.7], [0.3, 0.4]>}

And each diagnosis A_i (i=1, 2, 3) can also be represented by interval valued neutrosophic numbers with respect to all the symptoms as follows:

= {< x_1 , [0.5, 0.6], [0.2 ,0.3], [0.4, 0.5] >, < x_2 , [0.2 , 0.6], [0.3 ,0.4], [0.6 , 0.7]>,< x_3 , [0.1, 0.2], [0.3 ,0.6], [0.7, 0.8]>}

= {< x_1 , [0.4, 0.5], [0.3, 0.4], [0.5, 0.6] >, < x_2 , [0.3, 0.5], [0.4 ,0.6], [0.2, 0.4]> , < x_3 , [0.3, 0.6], [0.1, 0.2], [0.5, 0.6]>}

= {< x_1 , [0.6, 0.8], [0.4 ,0.5], [0.3, 0.4]>, < x_2 , [0.3, 0.7], [0.2, 0.3], [0.4, 0.7]> ,< x_3 , [0.3, 0.5], [0.4, 0.7], [0.2, 0.6]>}

Our aim is to classify the pattern P in one of the classes A_1 , A_2 , A_3 .According to the recognition principle of maximum degree of similarity measure between interval valued neutrosophic numbers, the process of diagnosis A_k to patient P is derived according to

k = arg Max{ $C_N(A_i, P)$ **)**}

from the previous formula (7) , we can compute the cosine similarity between A_i (i=1, 2, 3) and P as follows;

$C_N(A_1, P)$ **=0.8988,** $C_N(A_2, P)$ **=0.8560,** $C_N(A_3, P)$ **=0.9654**

Then, we can assign the patient to diagnosis A_3 (Typoid) according to recognition of principal.

VI. Conclusions.
In this paper a cosine similarity measure between two and weighted interval valued neutrosophic sets is proposed. The results of the proposed similarity measure and existing similarity measure are compared. Finally, the proposed cosine similarity measure is applied to pattern recognition.

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A Study on Problems of Hijras in West Bengal Based on Neutrosophic Cognitive Maps

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Abstract: This paper deals with the problems faced by Hijras in West Bengal in order to find its solutions using neutrosophic cognitive maps. Florentin Smarandache and Vasantha Kandasamy studied neutrosophic cognitive map which is an extension of fuzzy cognitive map by incorporating indeterminacy.Hijras is considered as neither man nor woman in biological point of view. They are in special gender identity (third gender) in Indian society. In their daily life, they have to face many of problems in social aspects. Some of the problems namely, absence of social security, education problem, bad habits, health problem, stigma and discrimination, access to information and service problem, violence, Hijra community issues, and sexual behavior problem are considered in this study. Based on the expert's opinion and the notion of indeterminacy, we formulate neutrosophic cognitive map. Then we studied the effect of two instantaneous state vectors separately on connection matrix E and neutrosophic adjacency matrix N(E).

Keywords: Fuzzy cognitive map, neutrosophic cognitive maps, indeterminacy, instantaneous state vector, Hijras

Introduction

The Hijra existence is deeply rooted in Indian culture. The great epic Ramayana references a third gender, neither male nor female, as individual whom Lord Rama blesses. Other Indian religious texts, including the Mahabharata, mention the additional examples of male deities adopting the female form and vice versa. It is important to notice that the ancient stories legitimize the Hijra existence and offer ample evidence of the profound spiritual connection with the Hijras. Ancient stories depicts that Hijras like to maintain their feminine identity. As a result, Hijras adhere to a strict, institutionalized code of conduct that defines the Hijra value system and way of life. Hijras are transgender male-to-female transitioned individuals. Their community does not include the individuals who change their sex from female to male or male to female.

The popular understanding of the Hijra as an alternative sex and gender is based on the model of the hermaphrodite, a person having both male and female sex organ.

The linguistic evidence suggests that Hijras are mainly thought of as more female than male. The word Hijra is a masculine noun, most widely translated into English as eunuch or hermaphrodite. Both these words reflect sexual impotence, which is understood in India to mean a physical defect impairing sexual function. It is widely believed in India that a man who has continued sexual relations in the receiver role will lose sexual vitality in his genitals and become impotent. It is sexual impotence (with women), then, and not sexual relations with men that defines the potential Hijra. Hijras identify themselves as incomplete men in the sense that they do not have desires for women that other men do. They attribute this lack of desire to a defective male sexual organ. Hijra role is defined biologically as a loss of virility, or as "man minus man". Thus, Indian emic sex and gender categories of Hijra collapse the tic categories of (born) hermaphrodites and eunuch. While ambiguous male genitalia serve as the most important culturally defined sign of the Hijra, in practical terms any indication of a loss of masculinity, whether impotence, effeminate behavior or desire for sexual relations with men in the receptor role, may be taken as a sign that one should join the Hijras.

Hijras are important part of our society. The central problem of a Hijra is the absence of social security. The other day to day problems are mental health, stigma and discrimination, access to information and services, violence, Hijra community issues, sexual behavior, and physical health problems. They are working under unsecured environment or work culture (short dance to take new born baby, clapping, biting dhol, collecting food, dresses forcefullyroo, etc. Sometimes they are seen in begging in train (local, passenger, express), begging in buses (local, express, long root). Some times they are seen snatching money bags or other things to protest the misbehavior of the passengers. They experience very inhumen situation in thier work place because of

misconception of the common people about the Hijras. On the other hand, Hijras are engaged in an old singing and dancing culture, acting, film producing, social activities etc.

Political scientist Robert Axelrod introduced the concept of cognitive maps (CMs) in 1976. He applied it in political science. Axelrod developed CMs, i.e. signed digraphs designed to capture the causal assertions of an individual with respect to a certain domain and then applied them in order to analyze the effects of alternatives, e.g. policies, business decisions, etc. upon certain goals. A cognitive map comprises of only two basic types of elements namely, nodes and edges. Nodes represent variable concepts and edges represent causal connections. In the field of cognitive maps, the concept of fuzzy set theory introduced by Zadeh was introduced by B. Kosko in 1986[3]. F. Smarandache and Vasantha Kandasamy [15] widely studied neutrosophic cognitive map (NCM) which is the extension of fuzzy cognitive map by incorporating indeterminacy. Chakrabarti and Pramanik [18] studied the problems of construction workers in West Bengal based on NCM. The concept of neutrosophic logic plays a vital role in several real life problems like law, information technology, stocks and share etc. Hijras' problem in West Bengal is one of the major problems in India. Nowadays, Hijras have to face many problems in their day to day life, although they are important part of our society. Some of the problems are discussed in the present study. The present study was done among 36 Hijras in West Bengal. Major problems of Hijras are absence of social security, mental health, stigma and discrimination, access to information and services, violence, regional issues, sexual behavior, and physical health problems. Rest of the paper is presented in the following way. Section II describes the preliminaries of NCM. Section III presents the method of finding hidden pattern. Section IV is devoted to present the modeling the problems of Hijras using NCM. Section V presents conclusions and future work.

Section II

Mathematical preliminaries:

Definition: 2.1 Neutrosophic graph: A Neutrosophic graph refers to a graph in which at least one edge is an indeterminacy denoted by dotted lines.

Definition: 2.2 Neutrosophic directed graph: A neutrosophic directed graph is a directed graph which has at least one edge to be indeterminacy.

Definition: 2.3 Neutrosophic oriented graph: A neutrosophic oriented graph refers to a neutrosophic directed graph having no symmetric pair of directed indeterminacy lines.

Definition: 2.4 Neutrosophic Cognitive Map (NCM): An NCM refers to a neutrosophic directed graph with concepts like policies, events etc. as nodes and causalities or indeterminate as edges. It reflects the causal relationship between concepts. Let us suppose that $C_1, C_2,\ldots, C_k$ represent k nodes. Also let each node be a neutrosophic vector from neutrosophic vector space V. So a node C_j,($j = 1, 2, \ldots, k$) can be represented by ($x_1, x_2, \ldots, x_k$) where x_i's are zero or one or I (I represents the indeterminacy) and $x_i = 1$ means that the node C_i is on state and $x_i = 0$ implies that the node is in the off state and $x_i = I$ means the node is an indeterminate state at that time or in that situation. Let C_m and C_n denote the two nodes of the NCM. The directed edge from C_m to C_n represents the causality of C_m on C_n called connections. Every edge in the NCM is weighted with a number in the set {-1, 0, 1, I}. Let α_{mn} denote the weight of the directed edge C_mC_n, $\alpha_{mn} \in \{-1, 0, 1, I\}$.

$\alpha_{mn}= 0$ if C_m does not have any effect on C_m,

$\alpha_{mn}= 1$ if increase (or decrease) in C_m causes increase (or decreases) in C_n, $\alpha_{mn}= -1$ if increase (or decrease) in C_m causes decrease (or increase) in C_n.

$\alpha_{mn}= I$ if the relation or effect of C_m on C_n is an indeterminate**.**

Definition: 2.5

NCMs with edge weight from the set {-1, 0, 1, I} are called simple NCMs.

Definition: 2.6

Let $C_1, C_2,\ldots, C_k$ be the nodes of a NCM. Let the neutrosophic matrix N(E) be defined as $N(E) =(\alpha_{mn})$, where α_{mn} is the weight of the directed edge C_mC_n, where $\alpha_{mn} \in \{-1, 0, 1, I\}$. N(E) is called the neutrosophic adjacency matrix of the NCM.

Definition: 2.7

Let $C_1, C_2, \ldots, C_k$ denote the nodes of the NCM. Let $A = (\alpha_1 ,..., \alpha_2 , \alpha_k)$, where $\alpha_m \in \{0, 1, I\}$. A is called the instantaneous state neutrosophic vector and it denotes the on – off – indeterminate state position of the node at an instant

$\alpha_m= 0$ if $\alpha_m$is off (no effect)

$\alpha_m= 1$ if $\alpha_m$is on (has effect

$\alpha_m= I$ if α_m is indeterminate (effect cannot be determined) for m = 1, 2,…, k.

Definition: 2.8

Let $C_1, C_2, \ldots, C_k$ be the nodes of the NCM. Let C_1C_2 , C_2C_3 , C_3C_4 , C_4C_5 , . . ., C_mC_n be the edges of the NCM. Then the edges constitute a directed cycle.

An NCM is said to be cyclic if it possesses a directed cyclic. An NCM is said to be acyclic if it does not possess any directed cycle.

Definition: 2.9

An NCM having cycles is said to have a feedback. When there exists a feedback in the NCM i.e. when the causal relations flow through a cycle in a revolutionary manner the NCM is termed a dynamical system.

Definition 2.10

Let C_1C_2, C_2C_3, $\overrightarrow{C_3C_4}$, $\overrightarrow{C_4C_5}$, . . ., $\overrightarrow{C_{k-1}C_k}$ be a cycle, when C_m is switched on and if the causality flows through the on and if the causality flows through the edges of a cycle and if it again causes C_m, then the dynamical system goes round and round. This is true for any node C_m, for m = 1, 2,…, k. The equilibrium state for this dynamical system is termed the hidden pattern.

Definition 2.11

If the equilibrium state of a dynamical system is a unique state vector, then it is called a fixed point. Consider the NCM with $C_1, C_2, \ldots, C_k$ as nodes.

For example, let us start the dynamical system by switching on C_1. Let us assume that the NCM settles down with C_1 and C_k on, i.e. the state vector remain as (1, 0,…, 1). This neutrosophic state vector (1, 0,…, 0, 1) is termed the fixed point.

Definition 2.12

If the NCM settles with a neutrosophic state vector repeating in the form:

$A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_m \rightarrow A_1$, then this equilibrium is called a limit cycle of the NCM.

Section III

Determining the Hidden Pattern

Let $C_1, C_2, \ldots, C_k$ be the nodes of an NCM with feedback. Let us assume that E be the associated adjacency matrix. We find the hidden pattern when C_1 is switched on when an input is provided as the vector $A_1 = (1, 0, 0,\ldots, 0)$, the data should pass through the neutrosophic matrix N(E), this is performed by multiplying A_1 by the matrix N(E). Let $A_1N(E)$

$=(\alpha_1, \alpha_2 \ldots. \alpha_n)$ with the threshold operation that is by replacing $\alpha_m$by 1 if $\alpha_m > p$ and α_m by 0 if $\alpha_m < p$ (p – a suitable positive integer) and α_m by 1 if α_m is not an integer. The resulting concept is updated; the concept C_1 is included in the updated vector by transforming the first coordinate as 1 in the resulting vector. Suppose $A_1N(E) \rightarrow A_2$, then consider $A_2N(E)$ and repeat the same procedure. The procedure is repeated till we get a limit cycle or a fixed point.

Section IV

Modeling of the problems of the Hijras in West Bengal using NCM

To assess the impact of problems faced by Hijras in the age group 14-45 years, data was collected from 36 Hijras in West Bengal. Based on linguistic questionnaire and the expert's opinion, we have considered the following concepts as $\{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9,\}$. The following nodes are considered as the main nodes for the problem.

C_1- Absence of social security:

Hijras are in lack of employment support, poor access to government welfare schemes, problems in accessing BPL cards, ration cards and in opening bank accounts. Maximum Hijras are in low income level, low social status and low family bonding. Their process of earning money is very uncertain.

C_2-Education problem: Hijras belong to the third sex. In their school life they have to face much mental harassment from other companions. They have minimum social sympathy and empathy.

C_3-Bad habits: It includes smoking (bidi, cigarate etc), consumption of pan masala, gutka and addiction of drugs. They clap anywhere for their special identity. They demonstrate odd behaviors such as indicating their undeveloped sex organ in public place, rebuking, using slangy language and expressions in public place when they are provoked.

C_4- Health problem: There is a lack of health services availability as well as accessibility. Stigma against these communities forced them to remain invisible most of the time.

C_5- Stigma and discrimination: It is observed that Hijras have to face stigma and discrimination in all walks of lives. There is a need to generate more advocacy material on these issues. Most people in larger society have little or no knowledge about Hijras. This resulted in myths, unfounded fears and stigma against them.

C_6- Access to information and service problem: There is lack of information about human rights and issues like sexual and reproductive health. In West Bengal, there is no scope of government service for Hijras till now.

C_7- Violence: There are cases of sexual harassment of Hijras by state related stake holders. Institutional violence: They have to face violence everywhere. Hijras are often physically forced into having unsafe sex. Larger community leaders often take irrational decisions against them.

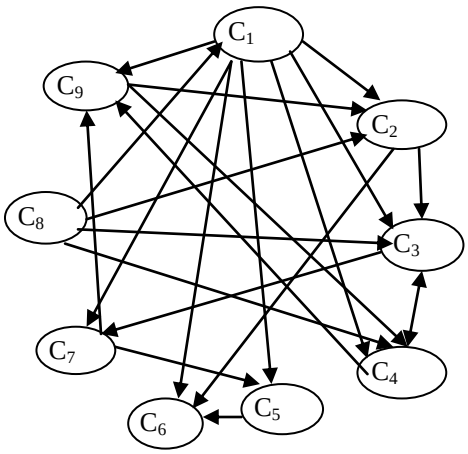
C_8- Hijra community issues: Hijras have closer knit community structures, but larger society is unaware about them. Hijra community leaders (Nayaks and Gurus) have total control over their communities. However, they do not necessarily possess information or the means for the development of their communities. The rigidity of their hierarchical community structure reflects that their Chelas (disciples or followers) could not question over their authority and suggest new ways of community development.

C_9- Sexual behavior problem: Hijras cannot enjoy normal sex life. Many women look like Hijras who spend their lives as sex workers. So there is a risk of HIV infection and vulnerability to HIV. Risk is based on

personal behavior, but vulnerability is related to the social environment in which one lives.

From NACO's point of view, targeted intervention programs focused on groups practicing high-risk sexual and other behaviors are the most important aspect. But in real life situation, everyone bear a unique and individual identity. These unique identities are closely related to a social position or situation for each person and each group of people. For Hijra sex workers, stigma and discrimination based on gender, sexuality and faith are part and parcel of their social situation, which increases their vulnerability to HIV. NACO has now acknowledged this situation. NACO is interested in observing how social inequities made each of these groups "differently vulnerable" to HIV. NACO also wants to make strategies in order to explore how these groups could be provided support in the form of safe spaces to combat the HIV epidemic. But pragmatic strategies can effectively made in order to deal the issues specific to Hijras if true picture of issues coming into light from the concerned Hijra community.

However more number of conceptual nodes can be added by the expert or investigator. Now we give the directed graph as well as neutrosophic directed graph in the following figures Fig.1 and Fig.2. Fig. 1 presents the directed graph with C_1, C_2,…,C_9 as nodes and Fig.2 presents the neutrosophic directed graph with the same nodes.



one way effect

two way effect

Fig. 1 Fuzzy cognitive map

The connection matrix E related to the graph in Fig.1 is given below:

$$E=\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

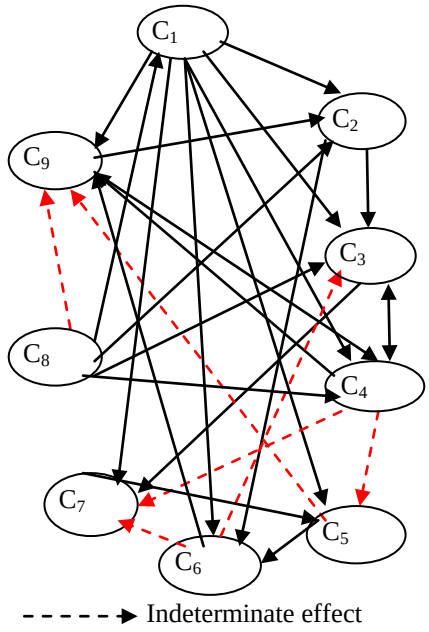


Fig. 2 Neutrosophicve map

The corresponding neutrosophic adjacency matrix N(E) related to the neutrosophic directed graph (see Fig.2) is given by the following matrix:

$$N(E)=\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & I & 0 & I & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & I \\ 0 & 0 & I & 0 & 0 & 0 & I & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & I \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Effect of two instantaneous state vectors separately on connection matrix E and Neutrosophic adjacency matrix N(E)

Case-I:

Suppose we take the instantaneous state vector A_1= (1 0 0 0 0 0 0 0 0)**,** the node "Absence of social security" is on state and all other nodes are off state.

At first, we study the effect of A_1 on E.

$A_1E=(0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1) \rightarrow (\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1) = A_2$

$A_2E=(0\ 2\ 3\ 3\ 1\ 2\ 2\ 0\ 3) \rightarrow (1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1) = A_3 = A_2$

According to the expert's opinion, the node "Hijra community issues" have no effect on the Hijras in absence of social security and vice versa and all other nodes are on state.

Now we study the effect of A_1 on N(E).

$A_1N(E)=(0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1) \rightarrow$

$(1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1) = A_2$

$A_2N(E)=(0\ 2\ 3{+}I\ 3\ I{+}1\ 3\ 2{+}2I\ 0\ 3{+}I) \rightarrow$

$(1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 1) = A_3 = A_2$

Thus according to the expert's opinion if C_1 is on state then the nodes C_2, C_3, C_4, C_5, C_6, C_7, C_9 are on state.

Case-II:

Again we take the state vector B_1

= (0 0 0 0 0 0 0 1 0), Hijra community issues (node) is on state and all other nodes are in off state. We will see the effect of B_1 on E and on N(E).

Now we find the effect of B_1=(0 0 0 0 0 0 0 1 0) on E.

$B_1E= (1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0) \rightarrow (1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0) = B_2$

$B_2E=(1\ 2\ 4\ 3\ 0\ 2\ 2\ 0\ 2) \rightarrow (1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1) = B_3$

$B_3E=(1\ 3\ 4\ 4\ 2\ 2\ 2\ 0\ 3) \rightarrow (1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 1) = B_4$

$= B_3$

Thus when the node "Hijra community issuses" is on state we see, "Stigma and discrimination" have no effect on the Hijras and all other nodes are on state.

Now we find the effect of B_1=(0 0 0 0 0 0 0 1 0) On M(E).

$B_1M(E)=(1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0) \rightarrow (1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 0) = B_2$

$B_2M(E)=(1\ 2\ 4\ 3\ 1{+}I\ 2\ 2{+}I\ 0\ 2{+}I) \rightarrow$

$(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = B_3$

$B_3M(E)=(1\ 3\ 4{+}I\ 4\ 1{+}I\ 3\ 3{+}2I\ 0\ 3{+}2I\ ) \rightarrow$

$(1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1) = B_4 = B_3$

Therefore, when the node C_8 is on state then the nodes C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_9 are on state. There is no nodes is on indeterminate state.

Conclusion

The problems of Hijras (Transgender) were studied based on NCM. It is noticed that if the Hijras (Transgender) are in social insecurity then they have to face educational problems and other factors like bad habits, health problem, access to information and service problem, violence, sexual behavior problems, stigma and discrimination.

Again, when regional issues increase or is on state, the following nodes namely, absence of social security, education problem, bad habits, health problem, access to information and service problem, stigma and discrimination, violence, sexual behavior problems will increase or are on states.

If new situation arises in the Hijras, new concepts need to be incorporated for modeling the problems of Hijras and that can be easily done by introducing new nodes.

Supreme Court of India recognizes Hijra (transgender) as 'third gender'(2014) [19]. However, government should implement the rights of Hijra (Transgender) and government should provide them education regarding their profession in order to avoid any unplesant and unexpected situations.

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NSS

Neutrosophic Crisp Set Theory

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Abstract. The purpose of this paper is to introduce new types of neutrosophic crisp sets with three types 1, 2, 3. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Also, we introduce and study the neutrosophic crisp point and neutrosophic crisp relations. Possible applications to database are touched upon.

Keywords: Neutrosophic Set, Neutrosophic Crisp Sets; Neutrosophic Crisp Relations; Generalized Neutrosophic Sets; Intuitionistic Neutrosophic Sets.

1 Introduction

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. The fundamental concepts of neutrosophic set, introduced by Smarandache in [16, 17, 18] and Salama et al. in [4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 19,20, 21], provides a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics. Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 4, 23] such as a neutrosophic set theory. In this paper we introduce new types of neutrosophic crisp set. After given the fundamental definitions and operations, we obtain several properties, and discussed the relationship between neutrosophic crisp sets and others. Also, we introduce and study the neutrosophic crisp points and relation between two new neutrosophic crisp notions. Finally, we introduce and study the notion of neutrosophic crisp relations.

2 Terminologies

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [16, 17, 18], and Salama et al. [7, 11, 12, 20]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0,1^{+}[$ is nonstandard unit interval.

Definition 2.1 [7]
A neutrosophic crisp set (NCS for short) $A=\langle A_1, A_2, A_3\rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3\rangle$ are subsets on X and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3\rangle$,

Salama et al. constructed the tools for developed neutrosophic crisp set, and introduced the NCS ϕ_N, X_N in X as follows:

ϕ_N may be defined as four types:

i) Type1: $\phi_N=\langle\phi,\phi,X\rangle$, or

ii) Type2: $\phi_N=\langle\phi,X,X\rangle$, or

iii) Type3: $\phi_N=\langle\phi,X,\phi\rangle$, or

iv) Type4: $\phi_N=\langle\phi,\phi,\phi\rangle$

1) X_N may be defined as four types

i) Type1: $X_N=\langle X,\phi,\phi\rangle$,

ii) Type2: $X_N=\langle X,X,\phi\rangle$,

iii) Type3: $X_N=\langle X,X,\phi\rangle$,

iv) Type4: $X_N=\langle X,X,X\rangle$,

Definition 2.2 [6, 7]

Let $A=\langle A_1, A_2, A_3\rangle$ a NCS on X , then the complement of the set A (A^c , for short) may be defined as three kinds

(C_1) Type1: $A^c=\langle A^c{}_1, A^c{}_2, A^c{}_3\rangle$,

(C_2) Type2: $A^c=\langle A_3, A_2, A_1\rangle$

(C_3) Type3: $A^c = \langle A_3, A^c{}_2, A_1 \rangle$

Definition 2.3 [6, 7]

Let X be a non-empty set, and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$, then we may consider two possible definitions for subsets ($A \subseteq B$)

($A \subseteq B$) may be defined as two types:

1) Type1: $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \subseteq B_2$ and $A_3 \supseteq B_3$ or
2) Type2: $A \subseteq B \Leftrightarrow A_1 \subseteq B_1, A_2 \supseteq B_2$ and $A_3 \supseteq B_3$.

Definition 2.5 [6, 7]

Let X be a non-empty set, and NCSs A and B in the form $A = \langle A_1, A_2, A_3 \rangle$, $B = \langle B_1, B_2, B_3 \rangle$ are NCSS Then

1) $A \cap B$ may be defined as two types:
   i. Type1: $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ or
   ii. Type2: $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$
2) $A \cup B$ may be defined as two types:
   i) Type1: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$ or
   ii) Type2: $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$.

3 Some Types of Neutrosophic Crisp Sets

We shall now consider some possible definitions for some types of neutrosophic crisp sets

Definition 3.1

The object having the form $A = \langle A_1, A_2, A_3 \rangle$ is called

1) **(Neutrosophic Crisp Set with Type 1)** If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$. (NCS-Type1 for short).
2) **(Neutrosophic Crisp Set with Type 2)** If satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$ and $A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$. (NCS-Type2 for short).
3) **(Neutrosophic Crisp Set with Type 3)** If satisfying $A_1 \cap A_2 \cap A_3 = \phi$ and $A_1 \cup A_2 \cup A_3 = X$. (NCS-Type3 for short).

Definition 3.3

1) **(Neutrosophic Set [9, 16, 17]):** Let X be a non-empty fixed set. A neutrosophic set (NS for short) A is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of membership function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \le \mu_A(x), \sigma_A(x), \nu_A(x) \le 1^+$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

2) **(Generalized Neutrosophic Set [8]):** Let X be a non-empty fixed set. A generalized neutrosophic (GNS for short) set A is an object having the form $A = \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0^- \le \mu_A(x), \sigma_A(x), \nu_A(x) \le 1^+$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \wedge \nu_A(x) \le 0.5$ and $0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$.

3) **(Intuitionistic Neutrosophic Set [22]).** Let X be a non-empty fixed set. An intuitionistic neutrosophic set A (INS for short) is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where $0.5 \le \mu_A(x), \sigma_A(x), \nu_A(x)$ and the functions satisfy the condition $\mu_A(x) \wedge \sigma_A(x) \le 0.5$, $\mu_A(x) \wedge \nu_A(x) \le 0.5$, $\sigma_A(x) \wedge \nu_A(x) \le 0.5$, and $^- 0 \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 2^+$. A neutrosophic crisp with three types the object $A = \langle A_1, A_2, A_3 \rangle$ can be identified to an ordered triple $\langle A_1, A_2, A_3 \rangle$ are subsets on X, and every crisp set in X is obviously a NCS having the form $\langle A_1, A_2, A_3 \rangle$.

Every neutrosophic set $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ on X is obviously on NS having the form $\langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$.

Remark 3.1

1) The neutrosophic set not to be generalized neutrosophic set in general.
2) The generalized neutrosophic set in general not intuitionistic NS but the intuitionistic NS is generalized NS.

Intuitionistic NS ⟶ Generalized NS ⟶ NS

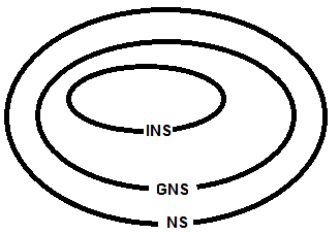

Fig. 1. Represents the relation between types of NS

Corollary 3.1

Let X non-empty fixed set and $A=\langle \mu_A(x),\sigma_A(x),\nu_A(x)\rangle$ be INS on X Then:

1) Type1- A^c of INS be a GNS.
2) Type2- A^c of INS be a INS.
3) Type3- A^c of INS be a GNS.

Proof

Since A INS then $\mu_A(x),\sigma_A(x),\nu_A(x)$, and

$\mu_A(x)\wedge\sigma_A(x)\le 0.5, \nu_A(x)\wedge\mu_A(x)\le 0.5$

$\nu_A(x)\wedge\sigma_A(x)\le 0.5$ Implies

$\mu^c{}_A(x),\sigma^c{}_A(x),\nu^c{}_A(x)\le 0.5$ then is not to be Type1- A^c INS. On other hand the Type 2- A^c,

$A^c=\langle \nu_A(x),\sigma_A(x),\mu_A(x)\rangle$ be INS and Type3- A^c,

$A^c=\langle \nu_A(x),\sigma^c{}_A(x),\mu_A(x)\rangle$ and $\sigma^c{}_A(x)\le 0.5$ implies to

$A^c=\langle \nu_A(x),\sigma^c{}_A(x),\mu_A(x)\rangle$ GNS and not to be INS

Example 3.1

Let $X=\{a,b,c\}$, and A,B,C are neutrosophic sets on X, $A=\langle 0.7,0.9,0.8)\setminus a,(0.6,0.7,0.6)\setminus b,(0.9,0.7,0.8\setminus c\rangle$,

$B=\langle 0.7,0.9,0.5)\setminus a,(0.6,0.4,0.5)\setminus b,(0.9,0.5,0.8\setminus c\rangle$

$C=\langle 0.7,0.9,0.5)\setminus a,(0.6,0.8,0.5)\setminus b,(0.9,0.5,0.8\setminus c\rangle$ By the Definition 3.3 no.3 $\mu_A(x)\wedge\sigma_A(x)\wedge\nu_A(x)\ge 0.5$, A be not GNS and INS,

$B=\langle 0.7,0.9,0.5)\setminus a,(0.6,0.4,0.5)\setminus b,(0.9,0.5,0.8\setminus c\rangle$ not INS, where $\sigma_A(b)=0.4<0.5$. Since

$\mu_B(x)\wedge\sigma_B(x)\wedge\nu_B(x)\le 0.5$ then B is a GNS but not INS.

$A^c=\langle 0.3,0.1,0.2)\setminus a,(0.4,0.3,0.4)\setminus b,(0.1,0.3,0.2\setminus c\rangle$

Be a GNS, but not INS.

$B^c=\langle 0.3,0.1,0.5)\setminus a,(0.4,0.6,0.5)\setminus b,(0.1,0.5,0.2\setminus c\rangle$

Be a GNS, but not INS, C be INS and GNS,

$C^c=\langle 0.3,0.1,0.5)\setminus a,(0.4,0.2,0.5)\setminus b,(0.1,0.5,0.2\setminus c\rangle$

Be a GNS but not INS.

Definition 3.2

A NCS-Type1 ϕ_{N_1}, X_{N_1} in X as follows:

1) ϕ_{N1} may be defined as three types:
   i) Type1: $\phi_{N_1}=\langle\phi,\phi,X\rangle$, or
   ii) Type2: $\phi_{N_1}=\langle\phi,X,\phi\rangle$, or
   iii) Type3: $\phi_N=\langle\phi,\phi,\phi\rangle$.
2) X_{N1} may be defined as one type

Type1: $X_{N_1}=\langle X,\phi,\phi\rangle$.

Definition 3.3

A NCS-Type2, ϕ_{N_2}, X_{N2} in X as follows:

1) ϕ_{N2} may be defined as two types:
   i) Type1: $\phi_{N_2}=\langle\phi,\phi,X\rangle$, or
   ii) Type2: $\phi_{N_2}=\langle\phi,X,\phi\rangle$
2) X_{N2} may be defined as one type

Type1: $X_{N_2}=\langle X,\phi,\phi\rangle$

Definition 3.4

A NCS-Type 3, ϕ_{N3}, X_{N3} in X as follows:

1) ϕ_{N3} may be defined as three types:
   i) Type1: $\phi_{N3}=\langle\phi,\phi,X\rangle$, or
   ii) Type2: $\phi_{N3}=\langle\phi,X,\phi\rangle$, or
   iii) Type3: $\phi_{N3}=\langle\phi,X,X\rangle$.
2) X_{N3} may be defined as three types
   i) Type1: $X_{N3}=\langle X,\phi,\phi\rangle$,
   ii) Type2: $X_{N3}=\langle X,X,\phi\rangle$,
   iii) Type3: $X_{N3}=\langle X,\phi,X\rangle$,

Corollary 3.2

In general

1- Every NCS-Type 1, 2, 3 are NCS.
2- Every NCS-Type 1 not to be NCS-Type2, 3.
3- Every NCS-Type 2 not to be NCS-Type1, 3.
4- Every NCS-Type 3 not to be NCS-Type2, 1, 2.
5- Every crisp set be NCS.

The following Venn diagram represents the relation between NCSs

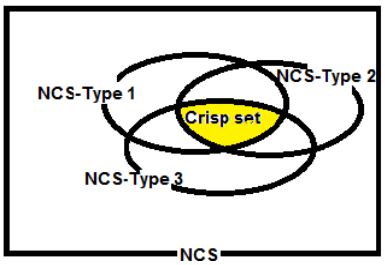

Fig 1. Venn diagram represents the relation between NCSs

Example 3.2

Let $X=\{a,b,c,d,e,f\}$, $A=\langle\{a,b,c,d\},\{e\},\{f\}\rangle$, $D=\langle\{a,b\},\{e,c\},\{f,d\}\rangle$ be a NCS-Type 2,

$B=\langle\{a,b,c\},\{d\},\{e\}\rangle$ be a NCT-Type1 but not NCS-Type 2, 3. $C=\langle\{a,b\},\{c,d\},\{e,f,a\}\rangle$ be a NCS-Type 3.but not NCS-Type1, 2.

Definition 3.5

Let X be a non-empty set, $A=\langle A_1,A_2,A_3\rangle$

1)If A be a NCS-Type1 on X , then the complement of the set A (A^c , for short) maybe defined as one kind of complement Type1: $A^c=\langle A_3,A_2,A_1\rangle$.

2) If A be a NCS-Type 2 on X , then the complement of the set A (A^c , for short) may be defined as one kind of complement $A^c=\langle A_3,A_2,A_1\rangle$.

3) If A be NCS-Type3 on X , then the complement of the set A (A^c , for short) maybe defined as one kind of complement defined as three kinds of complements

(C_1) Type1: $A^c=\langle A^c{}_1,A^c{}_2,A^c{}_3\rangle$,

(C_2) Type2: $A^c=\langle A_3,A_2,A_1\rangle$

(C_3) Type3: $A^c=\langle A_3,A^c{}_2,A_1\rangle$

Example 3.3

Let $X=\{a,b,c,d,e,f\}$, $A=\langle\{a,b,c,d\},\{e\},\{f\}\rangle$ be a NCS-Type 2, $B=\langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$ be a NCS-Type1., $C=\langle\{a,b\},\{c,d\},\{e,f\}\rangle$ NCS-Type 3, then the complement $A=\langle\{a,b,c,d\},\{e\},\{f\}\rangle$,

$A^c=\langle\{f\},\{e\},\{a,b,c,d\}\rangle$ NCS-Type 2, the complement of $B=\langle\{a,b,c\},\{\phi\},\{d,e\}\rangle$, $B^c=\langle\{d,e\},\{\phi\},\{a,b,c\}\rangle$ NCS-Type1. The complement of $C=\langle\{a,b\},\{c,d\},\{e,f\}\rangle$ may be defined as three types:

Type 1: $C^c=\langle\{c,d,e,f\},\{a,b,e,f\},\{a,b,c,d\}\rangle$.

Type 2: $C^c=\langle\{e,f\},\{a,b,e,f\},\{a,b\}\rangle$,

Type 3: $C^c=\langle\{e,f\},\{c,d\},\{a,b\}\rangle$,

Proposition 3.1

Let $\{A_j:j\in J\}$ be arbitrary family of neutrosophic crisp subsets on X, then

1) $\cap A_j$ may be defined two types as :

i) Type1: $\cap A_j=\langle\cap Aj_1,\cap A_{j_2},\cup A_{j_3}\rangle$,or

ii) Type2: $\cap A_j=\langle\cap Aj_1,\cup A_{j_2},\cup A_{j_3}\rangle$.

2) $\cup A_j$ may be defined two types as :

1) Type1: $\cup A_j=\langle\cup Aj_1,\cap A_{j_2},\cap A_{j_3}\rangle$ or

2) Type2: $\cup A_j=\langle\cup Aj_1,\cup A_{j_2},\cap A_{j_3}\rangle$.

Definition 3.6

(a) If $B=\langle B_1,B_2,B_3\rangle$ is a NCS in Y, then the preimage of B under f, denoted by $f^{-1}(B)$, is a NCS in X defined by $f^{-1}(B)=\langle f^{-1}(B_1),f^{-1}(B_2),f^{-1}(B_3)\rangle$.

(b) If $A=\langle A_1,A_2,A_3\rangle$ is a NCS in X, then the image of A under f, denoted by $f(A)$, is the a NCS in Y defined by $f(A)=\langle f(A_1),f(A_2),f(A_3)^c)\rangle$.

Here we introduce the properties of images and preimages some of which we shall frequently use in the following.

Corollary 3.3

Let A, $\{A_i:i\in J\}$, be a family of NCS in X, and B, $\{B_j:j\in K\}$ NCS in Y, and $f:X\to Y$ a function. Then

(a) $A_1\subseteq A_2\Leftrightarrow f(A_1)\subseteq f(A_2)$,

$B_1\subseteq B_2\Leftrightarrow f^{-1}(B_1)\subseteq f^{-1}(B_2)$,

(b) $A\subseteq f^{-1}(f(A))$ and if f is injective, then $A=f^{-1}(f(A))$,

(c) $f^{-1}(f(B))\subseteq B$ and if f is surjective, then $f^{-1}(f(B))=B$,

(d) $f^{-1}(\cup B_i)=f^{-1}(B_i)$, $f^{-1}(\cap B_i)=\cap f^{-1}(B_i)$,

(e) $f(\cup A_{ii})=\cup f(A_{ii})$; $f(\cap A_{ii})\subseteq\cap f(A_{ii})$; and if f is injective, then $f(\cap A_{ii})=\cap f(A_{ii})$;

(f) $f^{-1}(Y_N)=X_N$, $f^{-1}(\phi_N)=\phi_N$.

(g) $f(\phi_N)=\phi_N$, $f(X_N)=Y_N$, if f is subjective.

Proof

Obvious

4 Neutrosophic Crisp Points

One can easily define a nature neutrosophic crisp set in X, called "neutrosophic crisp point" in X, corresponding to an element X:

Definition 4.1

Let $A=\langle A_1,A_2,A_3\rangle$,be a neutrosophic crisp set on a set X, then $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, $p_1\neq p_2\neq p_3\in X$ is called a neutrosophic crisp point on A.

A NCP $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle$, is said to be belong to a neutrosophic crisp set $A=\langle A_1, A_2, A_3\rangle$, of X, denoted by $p \in A$, if may be defined by two types

Type 1: $\{p_1\}\subseteq A_1, \{p_2\}\subseteq A_2$ and $\{p_3\}\subseteq A_3$ or

Type 2: $\{p_1\}\subseteq A_1, \{p_2\}\supseteq A_2$ and $\{p_3\}\subseteq A_3$

Theorem 4.1

Let $A=\langle A_1, A_2, A_3\rangle$ and $B=\langle B_1, B_2, B_3\rangle$, be neutrosophic crisp subsets of X. Then $A\subseteq B$ iff $p\in A$ implies $p\in B$ for any neutrosophic crisp point p in X.

Proof

Let $A\subseteq B$ and $p\in A$, Type 1: $\{p_1\}\subseteq A_1, \{p_2\}\subseteq A_2$ and $\{p_3\}\subseteq A_3$ or Type 2: $\{p_1\}\subseteq A_1, \{p_2\}\supseteq A_2$ and $\{p_3\}\subseteq A_3$ Thus $p\in B$. Conversely, take any point in X. Let $p_1\in A_1$ and $p_2\in A_2$ and $p_3\in A_3$. Then p is a neutrosophic crisp point in X. and $p\in A$. By the hypothesis $p\in B$. Thus $p_1\in B_1$ or Type1: $\{p_1\}\subseteq B_1, \{p_2\}\subseteq B_2$ and $\{p_3\}\subseteq B_3$ or Type 2: $\{p_1\}\subseteq B_1, \{p_2\}\supseteq B_2$ and $\{p_3\}\subseteq B_3$. Hence $A\subseteq B$.

Theorem 4.2

Let $A=\langle A_1, A_2, A_3\rangle$, be a neutrosophic crisp subset of X. Then $A=\cup\{p : p\in A\}$.

Proof

Obvious

Proposition 4.1

Let $\{A_j : j\in J\}$ is a family of NCSs in X. Then

(a_1) $p=\langle\{p_1\},\{p_2\},\{p_3\}\rangle\in\bigcap_{j\in J}A_j$ iff $p\in A_j$ for each $j\in J$.

(a_2) $p\in\bigcup_{j\in J}A_j$ iff $\exists j\in J$ such that $p\in A_j$.

Proposition 4.2

Let $A=\langle A_1, A_2, A_3\rangle$ and $B=\langle B_1, B_2, B_3\rangle$ be two neutrosophic crisp sets in X. Then $A\subseteq B$ iff for each p we have $p\in A \Leftrightarrow p\in B$ and for each p we have $p\in A \Rightarrow p\in B$. iff $A=B$ for each p we have $p\in A \Rightarrow p\in B$ and for each p we have $p\in A \Leftrightarrow p\in B$.

Proposition 4.3

Let $A=\langle A_1, A_2, A_3\rangle$ be a neutrosophic crisp set in X. Then $A=\cup<\{p_1 : p_1\in A_1\},\{p_2 : p_2\in A_2\},\{p_3 : p_3\in A_3\}$.

Definition 4.2

Let $f: X\to Y$ be a function and p be a neutrosophic crisp point in X. Then the image of p under f, denoted by $f(p)$, is defined by $f(p)=\langle\{q_1\},\{q_2\},\{q_3\}\rangle$, where $q_1=f(p_1), q_2=f(p_2)$ and $q_3=f(p_3)$.It is easy to see that $f(p)$ is indeed a NCP in Y, namely $f(p)=q$, where $q=f(p)$, and it is exactly the same meaning of the image of a NCP under the function f.

Definition 4.3

Let X be a nonempty set and $p\in X$. Then the neutrosophic crisp point p_N defined by $p_N=\langle\{p\},\phi,\{p\}^c\rangle$ is called a neutrosophic crisp point (NCP for short) in X, where NCP is a triple ({only element in X}, empty set,{the complement of the same element in X }). Neutrosophic crisp points in X can sometimes be inconvenient when express neutrosophic crisp set in X in terms of neutrosophic crisp points. This situation will occur if $A=\langle A_1, A_2, A_3\rangle$ NCS-Type1, $p\notin A_1$. Therefore we shall define "vanishing" neutrosophic crisp points as follows:

Definition 4.4

Let X be a nonempty set and $p\in X$ a fixed element in X Then the neutrosophic crisp set $p_{N_N}=\langle\phi,\{p\},\{p\}^c\rangle$ is called vanishing" neutrosophic crisp point (VNCP for short) in X. where VNCP is a triple (empty set,{only element in X}, { the complement of the same element in X}).

Example 4.1

Let $X=\{a,b,c,d\}$ and $p=b\in X$. Then

$p_N=\langle\{b\},\phi,\{a,c,d\}\rangle$, $p_{N_N}=\langle\phi,\{b\},\{a,c,d\}\rangle$,

$P=\langle\{b\},\{a\},\{d\}\rangle$.

Now we shall present some types of inclusion of a neutrosophic crisp point to a neutrosophic crisp set:

Definition 4.5

Let $p_N=\langle\{p\},\phi,\{p\}^c\rangle$ is a NCP in X and $A=\langle A_1, A_2, A_3\rangle$ a neutrosophic crisp set in X.

(a) p_N is said to be contained in A ($p_N \in A$ for short) iff $p \in A_1$.

(b) p_{NN} be VNCP in X and $A = \langle A_1, A_2, A_3 \rangle$ a neutrosophic crisp set in X. Then p_{NN} is said to be contained in A ($p_{N_N} \in A$ for short) iff $p \notin A_3$.

Remark 4.2

p_N and p_{NN} are NCS-Type1

Proposition 4.4

Let $\{A_j : j \in J\}$ is a family of NCSs in X. Then

(a_1) $p_N \in \underset{j\in J}{\cap} A_j$ iff $p_N \in A_j$ for each $j \in J$.

(a_2) $p_{N_N} \in \underset{j\in J}{\cap} A_j$ iff $p_{N_N} \in A_j$ for each $j \in J$.

(b_1) $p_N \in \underset{j\in J}{\cup} A_j$ iff $\exists j \in J$ such that $p_N \in A_j$.

(b_2) $p_{N_N} \in \underset{j\in J}{\cap} A_j$ iff $\exists j \in J$ such that $p_{N_N} \in A_j$.

Proof

Straightforward.

Proposition 4.5

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ are two neutrosophic crisp sets in X. Then $A \subseteq B$ iff for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_N \in A \Rightarrow p_{N_N} \in B$. $A = B$ iff for each p_N we have $p_N \in A \Rightarrow p_N \in B$ and for each p_{N_N} we have $p_{N_N} \in A \Leftrightarrow p_{N_N} \in B$.

Proof

Obvious

Proposition 4.6

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then $A = (\cup \{p_N : p_N \in A\}) \cup (\cup \{p_{NN} : p_{NN} \in A\})$.

Proof

It is sufficient to show the following equalities:

$A_1 = (\cup \{p\} : p_N \in A\}) \cup (\cup \{\phi : p_{NN} \in A\})$, $A_3 = \phi$

and $A_3 = (\cap \{\{p\}^c : p_N \in A\}) \cap (\cap \{\{p\}^c : p_{NN} \in A\})$

which are fairly obvious.

Definition 4.6

Let $f : X \to Y$ be a function and p_N be a nutrosophic crisp point in X. Then the image of p_N under f, denoted by $f(p_N)$ is defined by $f(p_N) = \langle \{q\}, \phi, \{q\}^c \rangle$ where $q = f(p)$.

Let p_{NN} be a VNCP in X. Then the image of p_{NN} under f, denoted by $f(p_{NN})$, is defined by $f(p_{NN}) = \langle \phi, \{q\}, \{q\}^c \rangle$ where $q = f(p)$.

It is easy to see that $f(p_N)$ is indeed a NCP in Y, namely $f(p_N) = q_N$ where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under he function f. $f(p_{NN})$, is also a VNCP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p)$.

Proposition 4.7

States that any NCS A in X can be written in the form $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$, where $\underset{N}{A} = \cup \{p_N : p_N \in A\}$, $\underset{N}{A} = \phi_N$ and $\underset{NNN}{A} = \cup \{p_{NN} : p_{NN} \in A\}$. It is easy to show that, if $A = \langle A_1, A_2, A_3 \rangle$, then $\underset{N}{A} = \langle A_1, \phi, A_1^c \rangle$ and $\underset{NN}{A} = \langle \phi, A_2, A_3 \rangle$.

Proposition 4.8

Let $f : X \to Y$ be a function and $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then we have $f(A) = f(\underset{N}{A}) \cup f(\underset{NN}{A}) \cup f(\underset{NNN}{A})$.

Proof

This is obvious from $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$.

Proposition 4.9

Let $A = \langle A_1, A_2, A_3 \rangle$ and $B = \langle B_1, B_2, B_3 \rangle$ be two neutrosophic crisp sets in X. Then

a) $A \subseteq B$ iff for each p_N we have $p_N \in A \Leftrightarrow p_N \in B$ and for each p_{N_N} we have $p_N \in A \Rightarrow p_{N_N} \in B$.

b) $A = B$ iff for each p_N we have $p_N \in A \Rightarrow p_N \in B$ and for each p_{N_N} we have $p_{N_N} \in A \Leftrightarrow p_{N_N} \in B$.

Proof

Obvious

Proposition 4.10

Let $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then $A = (\cup \{p_N : p_N \in A\}) \cup (\cup \{p_{NN} : p_{NN} \in A\})$.

Proof

It is sufficient to show the following equalities:
$A_1 = \left(\cup \{p\} : p_N \in A\right) \cup \left(\cup \{\phi : p_{NN} \in A\}\right)$ $A_3 = \phi$
and $A_3 = \left(\cap \{p\}^c : p_N \in A\right) \cap \left(\cap \{p\}^c : p_{NN} \in A\right)$,
which are fairly obvious.

Definition 4.7

Let $f : X \to Y$ be a function.

(a) Let p_N be a neutrosophic crisp point in X. Then the image of p_N under f, denoted by $f(p_N)$, is defined by $f(p_N) = \langle \{q\}, \phi, \{q\}^c \rangle$, where $q = f(p)$.

(b) Let p_{NN} be a VNCP in X. Then the image of p_{NN} under f, denoted by $f(p_{NN})$, is defined by $f(p_{NN}) = \langle \phi, \{q\}, \{q\}^c \rangle$, where $q = f(p)$. It is easy to see that $f(p_N)$ is indeed a NCP in Y, namely $f(p_N) = q_N$, where $q = f(p)$, and it is exactly the same meaning of the image of a NCP under the function f. $f(p_{NN})$ is also a VNCP in Y, namely $f(p_{NN}) = q_{NN}$, where $q = f(p)$.

Proposition 4.11

Any NCS A in X can be written in the form $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$, where $\underset{N}{A} = \cup \{p_N : p_N \in A\}$, $\underset{N}{A} = \phi_N$ and $\underset{NNN}{A} = \cup \{p_{NN} : p_{NN} \in A\}$. It is easy to show that, if $A = \langle A_1, A_2, A_3 \rangle$, then $\underset{N}{A} = \langle x, A_1, \phi, A_1^c \rangle$ and $\underset{NN}{A} = \langle x, \phi, A_2, A_3 \rangle$.

Proposition 4.12

Let $f : X \to Y$ be a function and $A = \langle A_1, A_2, A_3 \rangle$ be a neutrosophic crisp set in X. Then we have
$f(A) = f(\underset{N}{A}) \cup f(\underset{NN}{A}) \cup f(\underset{NNN}{A})$.

Proof

This is obvious from $A = \underset{N}{A} \cup \underset{NN}{A} \cup \underset{NNN}{A}$.

5 Neutrosophic Crisp Set Relations

Here we give the definition relation on neutrosophic crisp sets and study of its properties.

Let X, Y and Z be three crisp nonempty sets

Definition 5.1

Let X and Y are two non-empty crisp sets and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ on X, $B = \langle B_1, B_2, B_3 \rangle$ on Y. Then

i) The product of two neutrosophic crisp sets A and B is a neutrosophic crisp set $A \times B$ given by

$A \times B = \langle A_1 \times B_1, A_2 \times B_2, A_3 \times B_3 \rangle$ on $X \times Y$.

ii) We will call a neutrosophic crisp relation $R \subseteq A \times B$ on the direct product $X \times Y$.

The collection of all neutrosophic crisp relations on $X \times Y$ is denoted as $NCR(X \times Y)$

Definition 5.2

Let R be a neutrosophic crisp relation on $X \times Y$, then the inverse of R is donated by R^{-1} where $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$ on $Y \times X$.

Example 5.1

Let $X = \{a,b,c,d\}$, $A = \langle \{a,b\}, \{c\}, \{d\} \rangle$ and $B = \langle \{a\}, \{c\}, \{d,b\} \rangle$ then the product of two neutrosophic crisp sets given by

$A \times B = \langle \{(a,a),(b,a)\}, \{(c,c)\}, \{(d,d),(d,b)\} \rangle$ and
$B \times A = \langle \{(a,a),(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$, and
$R_1 = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle$, $R_1 \subseteq A \times B$ on $X \times X$,
$R_2 = \langle \{(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$ $R_2 \subseteq B \times A$ on $X \times X$,
$R_1^{-1} = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle \subseteq B \times A$ and
$R_2^{-1} = \langle \{(b,a)\}, \{(c,c)\}, \{(d,d),(d,b)\} \rangle \subseteq B \times A$.

Example 5.2

Let $X = \{a,b,c,d,e,f\}$, $A = \langle \{a,b,c,d\}, \{e\}, \{f\} \rangle$,
$D = \langle \{a,b\}, \{e,c\}, \{f,d\} \rangle$ be a NCS-Type 2,
$B = \langle \{a,b,c\}, \{\phi\}, \{d,e\} \rangle$ be a NCS-Type1.
$C = \langle \{a,b\}, \{c,d\}, \{e,f\} \rangle$ be a NCS-Type 3.Then
$A \times D = \langle \{(a,a),(a,b),(b,a),(b,b),(b,b),(c,a),(c,b),(d,a),(d,b)\}, \{(e,e),(e,c)\}, \{(f,f),(f,d)\} \rangle$
$D \times C = \langle \{(a,a),(a,b),(b,a),(b,b)\}, \{(e,c),(e,d),(c,c),(c,d)\}, \{(f,e),(f,f),(d,e),(d,f)\} \rangle$
We can construct many types of relations on products.

We can define the operations of neutrosophic crisp relation.

Definition 5.3

Let R and S be two neutrosophic crisp relations between X and Y for every $(x,y) \in X \times Y$ and NCSS A and B in the form $A = \langle A_1, A_2, A_3 \rangle$ on X, $B = \langle B_1, B_2, B_3 \rangle$ on Y Then we can defined the following operations

i) $R \subseteq S$ may be defined as two types

a) Type1: $R \subseteq S \Leftrightarrow A_{1_R} \subseteq B_{1_S}$, $A_2 \subseteq B_2$, $A_{3R} \supseteq B_{3S}$

b) Type2: $R \subseteq S \Leftrightarrow A_{1_R} \subseteq B_{1_S}$, $A_{2R} \supseteq B_{2S}$, $B_{3S} \subseteq A_{3R}$

ii) $R \cup S$ may be defined as two types

a) Type1: $R \cup S = \langle A_{1R} \cup B_{1S}, A_{2R} \cup B_{2S}, A_{3R} \cap B_{3S} \rangle$,

b) Type2:

$R \cup S = \langle A_{1R} \cup B_{1S}, A_{2R} \cap B_{2S}, A_{3R} \cap B_{3S} \rangle$.

iii) $R \cap S$ may be defined as two types

a) Type1: $R \cap S = \langle A_{1R} \cap B_{1S}, A_{2R} \cup B_{2S}, A_{3R} \cup B_{3S} \rangle$,

b) Type2:

$R \cap S = \langle A_{1R} \cap B_{1S}, A_{2R} \cap B_{2S}, A_{3R} \cup B_{3S} \rangle$.

Theorem 5.1

Let R, S and Q be three neutrosophic crisp relations between X and Y for every $(x, y) \in X \times Y$, then

i) $R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}$.

ii) $(R \cup S)^{-1} \Rightarrow R^{-1} \cup S^{-1}$.

iii) $(R \cap S)^{-1} \Rightarrow R^{-1} \cap S^{-1}$.

iv) $\left(R^{-1}\right)^{-1} = R$.

v) $R \cap (S \cup Q) = (R \cap S) \cup (R \cap Q)$.

vi) $R \cup (S \cap Q) = (R \cup S) \cap (R \cup Q)$.

vii) If $S \subseteq R$, $Q \subseteq R$, then $S \cup Q \subseteq R$

Proof

Clear

Definition 5.4

The neutrosophic crisp relation $I \in NCR(X \times X)$, the neutrosophic crisp relation of identity may be defined as two types

i) Type1: $I = \{< \{A \times A\}, \{A \times A\}, \phi >\}$

ii) Type2: $I = \{< \{A \times A\}, \phi, \phi >\}$

Now we define two composite relations of neutrosophic crisp sets.

Definition 5.5

Let R be a neutrosophic crisp relation in $X \times Y$, and S be a neutrosophic crisp relation in $Y \times Z$. Then the composition of R and S, $R \circ S$ be a neutrosophic crisp relation in $X \times Z$ as a definition may be defined as two types

i) Type 1:

$R \circ S \leftrightarrow (R \circ S)(x, z) = \cup\{< \{(A_1 \times B_1)_R \cap (A_2 \times B_2)_S\}, \{(A_2 \times B_2)_R \cap (A_2 \times B_2)_S\}, \{(A_3 \times B_3)_R \cap (A_3 \times B_3)_S\} >$.

ii) Type 2:

$R \circ S \leftrightarrow (R \circ S)(x, z) = \cap\{< \{(A_1 \times B_1)_R \cup (A_2 \times B_2)_S\}, \{(A_2 \times B_2)_R \cup (A_2 \times B_2)_S\}, \{(A_3 \times B_3)_R \cup (A_3 \times B_3)_S\} >$.

Example 5.3

Let $X = \{a, b, c, d\}$, $A = \langle \{a, b\}, \{c\}, \{d\} \rangle$ and $B = \langle \{a\}, \{c\}, \{d, b\} \rangle$ then the product of two events given by $A \times B = \langle \{(a,a),(b,a)\}, \{(c,c)\}, \{(d,d),(d,b)\} \rangle$, and $B \times A = \langle \{(a,a),(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$, and

$R_1 = \langle \{(a,a)\}, \{(c,c)\}, \{(d,d)\} \rangle$, $R_1 \subseteq A \times B$ on $X \times X$,

$R_2 = \langle \{(a,b)\}, \{(c,c)\}, \{(d,d),(b,d)\} \rangle$ $R_2 \subseteq B \times A$ on $X \times X$.

$R_1 \circ R_2 = \cup \langle \{(a,a)\} \cap \{(a,b)\}, \{(c,c)\}, \{(d,d)\} \rangle$

$= \langle \{\phi\}, \{(c,c)\}, \{(d,d)\} \rangle$ and

$I_{A1} = \langle \{(a,a).(a,b).(b.a)\}, \{(a,a).(a,b).(b,a)\}, \{\phi\} \rangle$,

$I_{A2} = \langle \{(a,a).(a,b).(b.a)\}, \{\phi\}, \{\phi\} \rangle$

Theorem 5.2

Let R be a neutrosophic crisp relation in $X \times Y$, and S be a neutrosophic crisp relation in $Y \times Z$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

Proof

Let $R \subseteq A \times B$ on $X \times Y$ then $R^{-1} \subseteq B \times A$, $S \subseteq B \times D$ on $Y \times Z$ then $S^{-1} \subseteq D \times B$, from Definition 5.4 and similarly we can $I_{(R \circ S)^{-1}}(x, z) = I_{S^{-1}}(x, z)$ and $I_{R^{-1}}(x, z)$ then $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

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Interval Valued Fuzzy Neutrosophic Soft Structure Spaces

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Abstract. In this paper we introduce the topological structure of interval valued fuzzy neutrosophic soft sets and obtain some of its properties. We also investigate some operators of interval valued fuzzy neutrosophic soft topological space.

Keywords: Fuzzy Neutrosophic soft set, Interval valued fuzzy neutrosophic soft set, Interval valued fuzzy neutrosophic soft topological space.

1 Introduction

In 1999,[9] Molodsov initiated the novel concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty. In [6] Maji et al. intiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union , intersection, complement of fuzzy soft set. Moreover in [7,8] Maji et al extended soft sets to intuitionistic fuzzy soft sets and Neutrosophic soft sets.

Neutrosophic Logic has been proposed by Florentine Smarandache[14,15] which is based on non-standard analysis that was given by Abraham Robinson in 1960s. Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision undefined, incompleteness, inconsistency, redundancy, contradiction. The neutrosophic logic is a formal frame to measure truth, indeterminacy and falsehood. In Neutrosophic set, indeterminacy is quantified explicitly whereas the truth membership, indeterminacy membership and falsity membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors.

Yang et al.[16] presented the concept of interval valued fuzzy soft sets by combining the interval valued fuzzy set and soft set models. Jiang.Y et al.[5] introduced interval valued intuitionistic fuzzy soft set. In this paper we define interval valued fuzzy neutrosophic soft topological space and we discuss some of its properties.

2 Preliminaries

Definition 2.1[2]:

A fuzzy neutrosophic set A on the universe of discourse X is defined as

$A=\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$

where $T, I, F: X \rightarrow [0, 1]$ and

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

.

Definition 2.2[3]:

An interval valued fuzzy neutrosophic set (IVFNS in short) on a universe X is an object of the form $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ where $T_A(x) = X \rightarrow$ Int ([0,1]) , $I_A(x) = X \rightarrow$ Int ([0,1]) and $F_A(x) = X \rightarrow$ Int ([0,1]) {Int([0,1]) stands for the set of all closed subinterval of [0,1] satisfies the condition $\forall x \in X$, $\sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$.

Definition 2.3[3]:

Let U be an initial universe and E be a set of parameters. IVFNS(U) denotes the set of all interval valued fuzzy neutrosophic sets of U. Let $A \subseteq E$. A pair (F,A) is an interval valued fuzzy neutrosophic soft set over U, where F is a mapping given by $F: A \rightarrow$ IVFNS(U).

Note : Interval valued fuzzy neutrosophic soft set is denoted by IVFNS set.

Definition 2.4[3]:

The complement of an INFNSS (F,A) is denoted by $(F,A)^c$ and is defined as $(F,A)^c = (F^c, \neg A)$ where $F^c: \neg A \rightarrow$ IVFNSS(U) is a mapping given by $F^c(e) = \langle x, F_{F(\neg e)}(x), (I_{F(\neg e)}(x))^c, F_{F(\neg e)}(x)\rangle$ for all $x \in U$ and $e \in \neg A$, $(I_{F(\neg e)}(x))^c = 1 - I_{F(\neg e)}(x) = [1 - I_{F(e)}(x), 1 - \underline{I}_{F(e)}(x)]$.

Definition 2.5[3]:

The union of two IVFNSS (F,A) and (G,B) over a universe U is an IVFNSS (H,C) where $C = A \cup B, \forall e \in C$.

$$\tilde{H}_C(e) = \begin{cases} \left(\left\{ \dfrac{h}{T_{\tilde{F}(e)}(h), I_{\tilde{F}(e)}(h), F_{\tilde{F}(e)}(h)} \right\} \right), & \text{if } e \in A - B \\ \left(\left\{ \dfrac{h}{T_{\tilde{G}(e)}(h), I_{\tilde{G}(e)}(h), F_{\tilde{G}(e)}(h)} \right\} \right), & \text{if } e \in B - A \\ \left(\left\{ \dfrac{h}{T_{\tilde{H}(e)}(h), I_{\tilde{H}(e)}(h), F_{\tilde{H}(e)}(h)} \right\} \right), & \text{if } e \in A \cap B \end{cases}$$

where

$$T_{\tilde{H}(e)}(h) = \max\{T_{\tilde{F}(e)}(h), T_{\tilde{G}(e)}(h)\}$$
$$I_{\tilde{H}(e)}(h) = \max\{I_{\tilde{F}(e)}(h), I_{\tilde{G}(e)}(h)\},$$
$$F_{\tilde{H}(e)}(h) = \min\{F_{\tilde{F}(e)}(h), F_{\tilde{G}(e)}(h)\}$$

,

Definition 2.6[3]:

The intersection of two IVFNSS (F,A) and (G,B) over a universe U is an IVFNSS (H,C) where $C = A \cup B, \forall e \in C$.

$$\tilde{H}_C(e) = \begin{cases} \left(\left\{ \frac{h}{T_{\tilde{F}(e)}(h), I_{\tilde{F}(e)}(h), F_{\tilde{F}(e)}(h)} \right\} \right), & \text{if } e \in A - B \\ \left(\left\{ \frac{h}{T_{\tilde{G}(e)}(h), I_{\tilde{G}(e)}(h), F_{\tilde{G}(e)}(h)} \right\} \right), & \text{if } e \in B - A \\ \left(\left\{ \frac{h}{T_{\tilde{H}(e)}(h), I_{\tilde{H}(e)}(h), F_{\tilde{H}(e)}(h)} \right\} \right), & \text{if } e \in A \cap B \end{cases}$$

where

$$T_{\tilde{H}(e)}(h) = \min\{T_{\tilde{F}(e)}(h), T_{\tilde{G}(e)}(h)\}$$
$$I_{\tilde{H}(e)}(h) = \min\{I_{\tilde{F}(e)}(h), I_{\tilde{G}(e)}(h)\},$$
$$F_{\tilde{H}(e)}(h) = \max\{F_{\tilde{F}(e)}(h), F_{\tilde{G}(e)}(h)\}$$

,

3. INTERVAL VALUED FUZZY NEUTROSOPHIC SOFT TOPOLOGY

Definition 3.1:

Let (F_A, E) be an element of IVFNS set over (U,E),P(F_A, E) be the collection of all INFNS subsets of (F_A,E). A sub-family τ of P(F_A, E) is called an interval valued fuzzy neutrosophic soft topology (in short IVFNS-topology) on (F_A, E) if the following axioms are satisfied:

(i) (φ_A, E), (F_A, E) ∈τ.

(ii) $\{(f_A^k, E) / k \in K\} \subseteq \tau$ implies $\bigcup_{k \in K} (f_A^k, E) \in \tau$

(iii) If (f_A, E), (g_A, E) ∈τ then (f_A, E) ∩ (g_A, E) ∈τ.

Then the pair ((F_A, E),τ) is called interval valued fuzzy neutrosophic soft topological space (IVFNSTS). The members of τ are called τ-open IVFNS sets or open sets where φ_A:A→IVFNS(U) is defined as $\varphi_A(e)$ = {<x, [0,0],[0,0],[1,1]>:x∈U, ∀e∈A} and F_A:A→IVFNS(U) is defined as F_A (e) = {<x, [1,1],[1,1],[0,0]>:x∈U, ∀e∈A}.

Example 3.2:

Let U={h_1, h_2, h_3}, E = {e_1, e_2, e_3, e_4}, A = {e_1, e_2, e_3}.

(F_A, E) = {e_1 ={<h_1 [1,1], [1,1], [0,0]>,
<h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,
<h_3 [1,1], [1,1], [0,0]>}
{e_3 ={<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,
<h_3 [1,1], [1,1], [0,0]>}

(φ_A, E) ={e_1 ={<h_1 [0,0], [0,0], [1,1]>,
<h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}
{e_2 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A^1,E) = {e_1 ={<h_1 [0.5,0.6], [0.4,0.5], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.5,0.6], [0.0,0.1]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [0.4,0.5], [0.5,0.6], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.7,0.8], [0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A^2,E) = {e_1 ={<h_1 [0.3,0.4], [0.5,0.6], [0.1,0.2]>,
<h_2 [0.6,0.7], [0.5,0.6], [0.2,0.3]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [0.2,0.3], [0.4,0.5], [0.0,0.1]>,
<h_2 [1,1], [1,1], [0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A^3,E) = {e_1 ={<h_1 [0.3,0.4], [0.4,0.5], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.5,0.6], [0.2,0.3]>, <h_3 [1,1], [1,1], [0,0]>
{e_2 ={<h_1 [0.2,0.3], [0.4,0.5], [0.2,0.3]>,
<h_2 [0.4,0.5], [0.7,0.8], [0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}

(f_A^4,E) = {e_1 ={<h_1 [0.5,0.6], [0.5,0.6], [0.1,0.2]>,
<h_2 [0.6,0.7], [0.5,0.6], [0.0,0.1]>, <h_3 [1,1], [1,1], [0,0]>}
{e_2 ={<h_1 [0.4,0.5], [0.5,0.6], [0.0,0.1]>,
<h_2 [1,1], [1,1], [0,0]>, <h_3 [0,0], [0,0], [1,1]>}
{e_3 ={<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>,
<h_3 [0,0], [0,0], [1,1]>}.

Here τ={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E) ,(f_A^4,E)} of P(F_A, E) is a IVFNS topology on (F_A, E) and ((F_A, E), τ) is an interval valued fuzzy neutrosophic soft topological space.

Note: The subfamily τ_1={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E)} of P(F_A, E) is not an interval valued fuzzy neutrosophic soft topology on (F_A, E) since the union (f_A^1,E) ∪(f_A^2,E)= (f_A^4,E) does not belong to τ_1.

Definition 3.3:

As every IVFNS topology on (F_A, E) must contain the sets (φ_A, E) and (F_A, E), so the family τ={(φ_A, E), (F_A, E)} forms an IVFNS topology on (F_A, E). This topology is called indiscrete IVFNS- topology and the pair ((F_A, E), τ) is called an indiscrete interval valued fuzzy neutrosophic soft topological space.

Theorem 3.4:

Let {τ_i; i∈I} be any collection of IVFNS-topology on (F_A, E). Then their intersection $\bigcap_{i \in I} \tau_i$ is also a topology on (F_A, E).

Proof:

(i) Since $(\varphi_A, E), (F_A, E) \in \tau_i$ for each $i \in I$, hence $(\varphi_A, E), (F_A, E) \in \bigcap_{i \in I} \tau_i$.

(ii) Let $\{(f_A^k, E) / k \in K\}$ be an arbitrary family of interval valued fuzzy neutrosophic soft sets where $(f_A^k, E) \in \bigcap_{i \in I} \tau_i$ for each $k \in K$. Then for each $i \in I$, $(f_A^k, E) \in \tau_i$ for $k \in K$ and since for each $i \in I$, τ_i is an topology, therefore $\bigcup_{k \in K} (f_A^k, E) \in \tau_i$ for each $i \in I$. Hence $\bigcup_{k \in K} (f_A^k, E) \in \bigcap_{i \in I} \tau_i$.

(iii) Let $(f_A, E), (g_A, E) \in \bigcap_{i \in I} \tau_i$, then (f_A, E) and $(g_A, E) \in \tau_i$ for each $i \in I$ and since τ_i for each $i \in I$ is an topology, therefore $(f_A, E) \cap (g_A, E) \in \tau_i$ for each $i \in I$. Hnece $(f_A, E) \cap (g_A, E) \in \bigcap_{i \in I} \tau_i$ .Thus $\bigcap_{i \in I} \tau_i$ satisfies all the axioms of topology. Hence $\bigcap_{i \in I} \tau_i$ forms a topology.

But the union of topologies need not be a topology, which is shown in the following example.

Remark 3.5:

The union of two IVFNS – topology may not be a IVFNS- topology. If we consider the example 3.2 then the subfamilies $\tau_1 = \{(\varphi_A, E), (F_A, E), (f_A^1, E)\}$ and $\tau_2 = \{(\varphi_A, E), (F_A, E), (f_A^2, E)\}$ are the topologies in (F_A, E). But their union $\tau_1 \cup \tau_2 = \{(\varphi_A, E), (F_A, E), (f_A^1, E), (f_A^2, E)\}$ is not a topology on (F_A, E).

Definition 3.6:

Let $((F_A, E), \tau)$ be an IVFNS-topological space over (F_A, E). An IVFNS subset (f_A, E) of (F_A, E) is called interval valued fuzzy neutrosophic soft closed (IVFNS closed) if its complement $(f_A, E)^c$ is a member of τ.

Example 3.7:

Let us consider example 3.2, then the IVFNS closed sets in $((F_A, E), \tau)$ are

$(\varphi_A, E)^c = \{e_1 = \{\langle h_1 [1,1], [1,1], [0,0]\rangle, \langle h_2 [1,1], [1,1], [0,0]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$
$\{e_2 = \{\langle h_1 [1,1], [1,1], [0,0]\rangle, \langle h_2 [1,1], [1,1], [0,0]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$
$\{e_3 = \{\langle h_1 [1,1], [1,1], [0,0]\rangle, \langle h_2 [1,1], [1,1], [0,0]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$

$(F_A, E)^c = \{e_1 = \{\langle h_1 [0,0], [0,0], [1,1]\rangle, \langle h_2 [0,0], [0,0], [1,1]\rangle, \langle h_3 [0,0], [0,0], [1,1]\rangle\}$
$\{e_2 = \{\langle h_1 [0,0], [0,0], [1,1]\rangle, \langle h_2 [0,0], [0,0], [1,1]\rangle, \langle h_3 [0,0], [0,0], [1,1]\rangle\}$
$\{e_3 = \{\langle h_1 [0,0], [0,0], [1,1]\rangle, \langle h_2 [0,0], [0,0], [1,1]\rangle, \langle h_3 [0,0], [0,0], [1,1]\rangle\}$

$(f_A^1, E)^c = \{e_1 = \{\langle h_1 [0.2,0.3], [0.5,0.6], [0.5,0.6]\rangle, \langle h_2 [0.0,0.1], [0.4,0.5], [0.4,0.5]\rangle, \langle h_3 [0,0], [0,0], [1,1]\rangle\}$
$\{e_2 = \{\langle h_1 [0.2,0.3], [0.4,0.5], [0.4,0.5]\rangle, \langle h_2 [0,0], [0.2,0.3], [0.4,0.5]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$
$\{e_3 = \{\langle h_1 [1,1], [1,1], [0,0]\rangle, \langle h_2 [1,1], [1,1], [0,0]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$

$(f_A^2, E)^c = \{e_1 = \{\langle h_1 [0.1,0.2], [0.4,0.5], [0.3,0.4]\rangle, \langle h_2 [0.2,0.3], [0.4,0.5], [0.6,0.7]\rangle, \langle h_3 [0,0], [0,0], [1,1]\rangle\}$
$\{e_2 = \{\langle h_1 [0.0,0.1], [0.5,0.6], [0.2,0.3]\rangle, \langle h_2 [0,0], [0,0], [1,1]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$
$\{e_3 = \{\langle h_1 [1,1], [1,1], [0,0]\rangle, \langle h_2 [1,1], [1,1], [0,0]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$

$(f_A^3, E)^c = \{e_1 = \{\langle h_1 [0.2,0.3], [0.5,0.6], [0.3,0.4]\rangle, \langle h_2 [0.2,0.3], [0.4,0.5], [0.4,0.5]\rangle, \langle h_3 [0,0], [0,0], [1,1]\rangle\}$
$\{e_2 = \{\langle h_1 [0.2,0.3], [0.5,0.6], [0.2,0.3]\rangle, \langle h_2 [0,0], [0.2,0.3], [0.4,0.5]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$
$\{e_3 = \{\langle h_1 [1,1], [1,1], [0,0]\rangle, \langle h_2 [1,1], [1,1], [0,0]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$

$(f_A^4, E)^c = \{e_1 = \{\langle h_1 [0.1,0.2], [0.4,0.5], [0.5,0.6]\rangle, \langle h_2 [0.0,0.1], [0.4,0.5], [0.6,0.7]\rangle, \langle h_3 [0,0], [0,0], [1,1]\rangle\}$
$\{e_2 = \{\langle h_1 [0.0,0.1], [0.4,0.5], [0.4,0.5]\rangle, \langle h_2 [0,0], [0,0], [1,1]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$
$\{e_3 = \{\langle h_1 [1,1], [1,1], [0,0]\rangle, \langle h_2 [1,1], [1,1], [0,0]\rangle, \langle h_3 [1,1], [1,1], [0,0]\rangle\}$ are the interval valued fuzzy neutrosophic soft closed sets in $((F_A, E), \tau)$.

Theorem 3.8:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space over (F_A, E). Then

(i) $(\varphi_A, E)^c$ $(F_A, E)^c$ are interval valued fuzzy neutrosophic soft closed sets.

(ii) The arbitrary intersection of interval valued fuzzy neutrosophic soft closed sets is interval valued fuzzy neutrosophic soft closed set.

(iii) The union of two interval valued fuzzy neutrosophic soft closed sets is an interval valued fuzzy neutrosophic closed set.

Proof:

(i) Since $(\varphi_A, E), (F_A, E) \in \tau$ implies $(\varphi_A, E)^c$ and $(F_A, E)^c$ are closed.

(ii) Let $\{(f_A^k, E)/k \in K\}$ be an arbitrary family of IVFNS closed sets in $((F_A, E),\tau)$ and let $(f_A, E) = \bigcap_{k\in K}(f_A^k, E)$. Now $(f_A, E)^c = \left(\bigcap_{k\in K}(f_A^k, E)\right)^c = \bigcup_{k\in K}(f_A^k, E)^c$ and $(f_A^k, E)^c \in \tau$ for each $k\in K$, so $\bigcup_{k\in K}(f_A^k, E)^c \in \tau$. Hence $(f_A, E)^c \in \tau$. Thus (f_A, E) is IVFNS closed set.

(iii) Let $\{f_A^i, E)/i = 1,2,3,..n\}$ be a finite family of IVFNS closed sets in $((F_A, E),\tau)$ and let $(g_A, E) = \bigcup_{i=1}^{n}(f_A^i, E)$.

Now $(g_A, E)^c = \left(\bigcup_{i=1}^{n}(f_A^i, E)\right)^c = \bigcap_{i=1}^{n}(f_A^i, E)^c$ and $(f_A^i, E)^c \in \tau$. So $\bigcap_{i=1}^{n}(f_A^i, E)^c \in \tau$. Hence $(g_A, E)^c \in \tau$. Thus (g_A, E) is an IVFNS closed set.

Remark 3.9:

The intersection of an arbitrary family of IVFNS – open set may not be an IVFNS- open and the union of an arbitrary family of IVFNS closed set may not be an IVFNS closed.

Let us consider U= $\{h_1,h_2.h_3\}$; E = $\{e_1,e_2,e_3,e_4\}$,
A = $\{e_1,e_2,e_3\}$ and let
$(F_A, E) = \{e_1 = \{<h_1\ [1,1], [1,1], [0,0]>,$
$<h_2\ [1,1], [1,1], [0,0]>, <h_3\ [1,1], [1,1], [0,0]>\}$
$\{e_2 = \{<h_1\ [1,1], [1,1], [0,0]>, <h_2\ [1,1], [1,1], [0,0]>,$
$<h_3\ [1,1], [1,1], [0,0]>\}$
$\{e_3 = \{<h_1\ [1,1], [1,1], [0,0]>, <h_2\ [1,1], [1,1], [0,0]>,$
$<h_3\ [1,1], [1,1], [0,0]>\}$
$(\varphi_A, E) = \{e_1 = \{<h_1\ [0,0], [0,0], [1,1]>,$
$<h_2\ [0,0], [0,0], [1,1]>, <h_3\ [0,0], [0,0], [1,1]>\}$
$\{e_2 = \{<h_1\ [0,0], [0,0], [1,1]>, <h_2\ [0,0], [0,0], [1,1]>,$
$<h_3\ [0,0], [0,0], [1,1]>\}$
$\{e_3 = \{<h_1\ [0,0], [0,0], [1,1]>, <h_2\ [0,0], [0,0], [1,1]>,$
$<h_3\ [0,0], [0,0], [1,1]>\}$
For each $n\in N$, we define

$$(f_A^n, E) = \left\{ e_1 = \left\{ \left\langle h_1\left[\frac{1}{4n},\frac{1}{2n}\right],\left[\frac{1}{5n},\frac{1}{2n}\right],\left[\frac{1}{2}-\frac{1}{2n},\frac{1}{2}-\frac{1}{3n}\right]\right\rangle, \left\langle h_2[1,1],[1,1],[0,0]\right\rangle, \left\langle h_3[0,0],[0,0],[1,1]\right\rangle \right\} \right.$$

$$e_2 = \left\{ \left\langle h_1\left[\frac{1}{3n},\frac{1}{2n}\right],\left[\frac{1}{4n},\frac{1}{2n}\right],\left[\frac{1}{3}-\frac{1}{3n},\frac{1}{3}-\frac{1}{4n}\right]\right\rangle, \left\langle h_2[0,0],[0,0],[1,1]\right\rangle, \left\langle h_3[0,0],[0,0],[1,1]\right\rangle \right\}$$

$$\left. e_3 = \left\{ \left\langle h_1[0,0],[0,0],[1,1]\right\rangle, \left\langle h_2[0,0],[0,0],[1,1]\right\rangle, \left\langle h_3[0,0],[0,0],[1,1]\right\rangle \right\} \right\}$$

We observe that $\tau = \{(F_A, E), (\varphi_A, E), (f_A^n, E)\}$ is a IVFNS topology on (F_A, E).

But $\bigcap_{n=1}^{\infty}(f_A^n, E) = \{e_1 = \{<h_1\ [0,0], [0,0], [0.5,0.5]>,$
$<h_2\ [1,1],[1,1],[0,0]>, <h_3\ [0,0], [0,0], [1,1]>\}$
$\{e_2 = \{<h_1\ [0,0], [0,0], [0.33,0.33]>, <h_2\ [0,0], [0,0], [1,1]>,$
$<h_3\ [0,0], [0,0], [1,1]>\}$
$\{e_3 = \{<h_1\ [0,0], [0,0], [1,1]>, <h_2\ [0,0], [0,0], [1,1]>,$
$<h_3\ [0,0], [0,0], [1,1]>\}\}$ is not an IVFNS-open set in IVFNS topological space $((F_A, E),\tau)$, since $\bigcap_{n=1}^{\infty}(f_A^n, E) \notin \tau$.

The IVFNS closed sets in the IVFNS topological space $((F_A, E),\tau)$ are $(F_A, E)^c$,$(\varphi_A, E)^c$ and $(f_A^n, E)^c$ for (n= 1,2,3…).

But $\bigcup_{n=1}^{\infty}(f_A^n, E) = \{e_1 = \{<h_1\ [0.5,0.5], [1,1], [0,0]>,$
$<h_2\ [0,0],[0,0],[1,1]>, <h_3\ [1,1], [1,1], [0,0]>\}$
$\{e_2 = \{<h_1[0.33,0.33], [1,1], [0,0]>, <h_2\ [1,1], [1,1], [0,0]>,$
$<h_3\ [1,1], [1,1], [0,0]>\}$
$\{e_3 = \{<h_1\ [1,1], [1,1], [0,0]>, <h_2\ [1,1], [1,1], [0,0]>,$
$<h_3\ [1,1], [1,1], [0,0]>\}$ is not an IVFNS-closed set in IVFNS topological space $((F_A, E),\tau)$, since $\bigcup_{n=1}^{\infty}(f_A^n, E) \notin \tau$.

Definition 3.10:

Let $((F_A, E), \tau_1)$ and $((F_A, E), \tau_2)$ be two IVFNS topological spaces. If each $(f_A,E) \in \tau_1$ implies $(f_A,E) \in \tau_2$, then τ_2 is called interval valued fuzzy neutrosophic soft finer topology than τ_1 and τ_1 is called interval valued fuzzy neutrosophic soft coarser topology than τ_2.

Example 3.11:

If we consider the topologies τ_1={(φ_A, E), (F_A, E), (f_A^1 ,E), (f_A^2 ,E), (f_A^3 ,E) ,(f_A^4 ,E)} as in example 3.2 and τ_2={(φ_A, E), (F_A, E), (f_A^1,E),(f_A^3,E)} on (F_A,E). Then τ_1 is interval valued fuzzy neutrosophic soft finer than τ_2 and τ_2 is interval valued fuzzy neutrosophic soft coarser topology than τ_1.

Definition 3.12:

Let ((F_A, E), τ) be an IVFNS topological space of (F_A, E) and $\mathcal{B}$ be a subfamily of τ. If every element of τ can be expressed as the arbitrary interval valued fuzzy neutrosophic soft union of some element of $\mathcal{B}$, then $\mathcal{B}$ is called an interval valued fuzzy neutrosophic soft basis for the interval valued fuzzy neutrosophic soft topology τ.

Example 3.13:

In example 3.2 for the topology τ={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2 ,E), (f_A^3 ,E) ,(f_A^4 ,E)} the subfamily $\mathcal{B}$ ={(φ_A, E), (F_A, E), (f_A^1,E), (f_A^2 ,E), (f_A^3 ,E) } of P(F_A,E) is a basis for the topology τ.

Definition 3.14:

Let τ be the IVFNS topology on (F_A,E) $\in$ IVFNS(U,E) and (f_A,E) be an IVFNS set in P(F_A,E) is a neighborhood of a IVFNS set (g_A,E) if and only if there exist an τ-open IVFNS set (h_A,E) ie., (h_A,E)$\in\tau$ such that (g_A,E)$\subseteq$ (h_A,E)$\subseteq$ (f_A,E).

Example 3.15:

Let U={h_1, h_2, h_3}, E = {e_1, e_2, e_3, e_4}, A = {e_1}.In an IVFNS topology τ={(φ_A, E), (F_A, E), (h_A,E)} where
(F_A, E) = {e_1 ={<h_1 [1,1], [1,1], [0,0]>,
<h_2 [1,1], [1,1], [0,0]>, <h_3 [1,1], [1,1], [0,0]>}
(φ_A, E) = {e_1 ={<h_1 [0,0], [0,0], [1,1]>,
<h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>}
(h_A, E) = {e_1 ={<h_1 [0.4,0.5], [0.5,0.6], [0.4,0.5]>,
<h_2 [0.3,0.4], [0.4,0.5], [0.5,0.6]>,
<h_3 [0.4,0.5], [0.3,0.4], [0.1,0.2]>}}.
The IVFNS set
(f_A, E) = {e_1 ={<h_1 [0.5,0.6], [0.6,0.7], [0.2,0.3]>,
<h_2 [0.3,0.4], [0.4,0.5], [0.5,0.6]>,
<h_3 [0.4,0.5], [0.4,0.5], [0,0.1]>} is a neighbourhood of the IVFNS set
(g_A, E) = {e_1 ={<h_1 [0.3,0.4], [0.4,0.5], [0.4,0.5]>,
<h_2 [0.1,0.2], [0.2,0.3], [0.6,0.7]>,
<h_3 [0.4,0.5], [0.2,0.3], [0.3,0.4]>}}
because there exist an τ-open IVFNS set (h_A,E) such that (g_A,E)$\subseteq$ (h_A,E)$\subseteq$ (f_A,E).

Theorem 3.16:

A IVFNS set (f_A,E) in P(F_A,E) is an open IVFNS set if and only if (f_A,E) is a neighbourhood of each IVFNS set (g_A,E) contained in (f_A,E).

Proof:

Let (f_A,E) be an open IVFNS set and (g_A,E) be any IVFNS set contained in (f_A,E). Since we have (g_A,E)$\subseteq$ (h_A,E)$\subseteq$ (f_A,E) , it follows that (f_A,E) is a neighbourhood of (g_A,E). Conversely let (f_A,E) be a neighbourhood for every IVFNS set contained in it. Since (f_A,E)$\subseteq$(f_A,E) there exist an open IVFNS set (h_A,E) such that (f_A,E)$\subseteq$ (h_A,E)$\subseteq$ (f_A,E). Hence (h_A,E) = (f_A,E) and (f_A,E) is open.

Definition 3.17:

Let ((F_A, E), τ) be an interval valued fuzzy neutrosophic soft topological space on (F_A, E) and (f_A,E) be a IVFNS set in P(F_A,E). The family of all neighbourhoods of (f_A,E) is called the neighbourhood system of (f_A,E) up to topology and is denoted by $N_{(f_A,E)}$.

Theorem 3.18:

Let ((F_A, E), τ) be an interval valued fuzzy neutrosophic soft topological space. If $N_{(f_A,E)}$ is the neighbourhood system of an IVFNS set (f_A,E). Then
(i) Finite intersections of members of $N_{(f_A,E)}$ belong to $N_{(f_A,E)}$
(ii) Each interval valued fuzzy neutrosophic soft set which contains a member of $N_{(f_A,E)}$ belongs to $N_{(f_A,E)}$

Proof:

(i) Let (g_A,E) and (h_A,E) are two neighbourhoods of (f_A,E) , so there exist two open sets (g'_A, E) , (h'_A, E) such that (f_A,E)$\subseteq$ (g'_A, E) $\subseteq$(g_A,E) and (f_A,E)$\subseteq$ (h'_A, E) $\subseteq$(h_A,E).

Hence (f_A,E)$\subseteq$ $(g'_A, E) \cap (h'_A, E) \subseteq$ (g_A,E)$\cap$ (h_A,E) and $(g'_A, E) \cap (h'_A, E)$ is open. Thus (g_A,E)$\cap$ (h_A,E) is a neighbourhood of (f_A,E).

(ii) Let (g_A,E) is a neighbourhood of (f_A,E) and (g_A,E)$\subseteq$ (h_A,E), so there exist an open set (g^*_A, E) , such that (f_A,E)$\subseteq$ (g^*_A, E) $\subseteq$(g_A,E). By hypothesis (g_A,E)$\subseteq$ (h_A,E), so (f_A,E)$\subseteq$ (g^*_A, E) $\subseteq$(g_A,E)$\subseteq$ (h_A,E) which implies that (f_A,E)$\subseteq$ (g^*_A, E) $\subseteq$(h_A,E) and hence (h_A,E) is a neighbourhood of (f_A,E).

Definition 3.19:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A, E) and (f_A,E) , (g_A,E) be IVFNS sets in $P(F_A,E)$ such that $(g_A,E) \subseteq (f_A,E)$. Then (g_A,E) is called an interior IVFNS set of (f_A,E) if and only if (f_A,E) is a neighbourhood of (g_A,E).

Definition 3.20:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in $P(F_A,E)$. Then the union of all interior IVFNS set of (f_A,E) is called the interior of (f_A,E) and is denoted by $int(f_A,E)$ and defined by $int(f_A,E) = \cup\{(g_A,E)/ (f_A,E)$ is a neighbourhood of $(g_A,E)\}$

Or equivalently $int(f_A,E)=\cup\{ (g_A,E)/ (g_A,E)$ is an IVFNS open set contained in $(f_A,E)\}$.

Example 3.21:

Let us consider the IVFNS topology $\tau=\{(\varphi_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E) ,(f_A^4,E)\}$ as in example 3.2 and let

$(f_A, E) = \{e_1=\{<h_1 [0.4,0.5], [0.6,0.7], [0.1,0.2]>, <h_2 [0.7,0.8], [0.6,0.7], [0.1,0.2]>, <h_3 [1,1], [1,1], [0,0]>\}$

$\{e_2 =\{<h_1 [0.3,0.4], [0.5,0.6], [0,0.1]>, <h_2 [1,1], [1,1], [0,0]>, <h_3 [0,0], [0,0], [1,1]>\}$

$\{e_3 =\{<h_1 [0,0], [0,0], [1,1]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [0,0], [0,0], [1,1]>\}$ be an IVFNS set.

Then $int(f_A,E) = \cup\{ (g_A,E)/ (g_A,E)$ is an IVFNS open set contained in $(f_A,E)\} = (f_A^2,E) \cup (f_A^3,E) =(f_A^2,E)$.

Since $(f_A^2,E) \subseteq (f_A,E)$ and $(f_A^3,E) \subseteq (f_A,E)$.

Theorem 3.22:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in $P(F_A,E)$. Then

(i) $int(f_A,E)$ is an open and $int(f_A,E)$ is the largest open IVFNS set contained in (f_A,E).

(ii) The IVFNS set (f_A,E) is open if and only if $(f_A,E) = int(f_A,E)$.

Proof: Proof follows form the definition.

Proposition 3.23:

For any two IVFNS sets (f_A,E) and (g_A,E) is an interval valued fuzzy neutrosophic soft topological space $((F_A, E), \tau)$ on $P(F_A, E)$ then

(i) $(g_A,E) \subseteq (f_A,E)$ implies $int(g_A,E) \subseteq int(f_A,E)$.

(ii) $int(\varphi_A,E) = (\varphi_A,E)$ and $int(F_A,E) = (F_A,E)$.

(iii) $int(int(f_A,E)) = int(f_A,E)$.

(iv) $int((g_A,E) \cap (f_A,E)) = int(g_A,E) \cap int(f_A,E)$.

(v) $int((g_A,E) \cup (f_A,E)) \supseteq int(g_A,E) \cup int(f_A,E)$.

Proof:

(i) Since $(g_A,E) \subseteq (f_A,E)$ implies all the IVFNS – open set contained in (g_A,E) also contained in (f_A,E). Therefore $\{(g_A^*,E) / (g_A^*,E)$ is an IVFNS open set contained in $(g_A,E)\} \subseteq \{ (f_A^*,E) / (f_A^*,E)$ is an IVFNS open set contained in $(f_A,E)\}$. So $int(g_A,E) \subseteq int(f_A,E)$.

(ii) Proof is obvious.

(iii) $int(int(f_A,E)) = \cup\{ (g_A,E)/ (g_A,E)$ is an IVFNS open set contained in $int(f_A,E)\}$ and since $int(f_A,E)$ is the largest open IVFNS sset contained in $int(f_A,E)$, Therefore $int(int(f_A,E)) = int(f_A,E)$.

(iv) Since $int(g_A,E) \subseteq (g_A,E)$ and $int(f_A,E) \subseteq (f_A,E)$, we have $int(g_A,E) \cap int(f_A,E) \subseteq (g_A,E) \cap (f_A,E)$----(1) Again since $(g_A,E) \cap (f_A,E) \subseteq (g_A,E)$ and $(g_A,E) \cap (f_A,E) \subseteq (f_A,E)$ we have $int((g_A,E) \cap (f_A,E)) \subseteq int(g_A,E)$ and $int((g_A,E) \cap (f_A,E)) \subseteq int(f_A,E)$. Therefore $int((g_A,E) \cap (f_A,E)) \subseteq int(g_A,E) \cap int(f_A,E)$ ------(2). From (1) and (2) $int((g_A,E) \cap (f_A,E)) = int(g_A,E) \cap int(f_A,E)$.

(v) Since $(g_A,E) \subseteq (g_A,E) \cup (f_A,E)$ and $(f_A,E) \subseteq (g_A,E) \cup (f_A,E)$ so $int(g_A,E) \subseteq int((g_A,E) \cup (f_A,E))$ and $int(f_A,E) \subseteq int((g_A,E) \cup (f_A,E))$. Hence $int(g_A,E) \cup int(f_A,E) \subseteq int((g_A,E) \cup (f_A,E)$.

Definition 3.24:

Let $((F_A, E), \tau)$ be an IVFNS topological space on (F_A,E) and let (f_A,E), (g_A,E) be two IVFNS set in $P(F_A,E)$. Then (g_A,E) is called an exterior IVFNS set of (f_A,E) if and only if (g_A,E) is an interior IVFNS set of the complement (f_A,E).

Definition 3.25:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in $P(F_A,E)$. Then the union of all exterior IVFNS set of (f_A,E) is called the exterior of (f_A,E) and is denoted by $ext(f_A,E)$ and is defined by $ext(f_A,E) = \cup\{ (g_A,E) / (f_A,E)^c$ is a neighbourhood of $(g_A,E)\}$. That is from definition $ext(f_A,E) = int((f_A,E)^c)$.

Proposition 3.26:

For any two IVFNS sets (f_A,E) and (g_A,E) in an interval valued fuzzy neutrosophic soft topological space $((F_A, E), \tau)$ on $P(F_A,E)$ then

(i) $ext(f_A,E)$ is open and is the largest open set contained in $(f_A,E)^c$.

(ii) $(f_A,E)^c$ is open if and only if $(f_A,E)^c = ext(f_A,E)$.

(iii) $(g_A,E) \subseteq (f_A,E)$ implies $ext(f_A,E) \subseteq ext(g_A,E)$.

(iv) $ext((g_A,E) \cap (f_A,E)) \supseteq ext(g_A,E) \cup ext(f_A,E)$.

(v) $ext((g_A,E) \cup (f_A,E)) = ext(g_A,E) \cap ext(f_A,E)$.

Proof:

Proofs are straight forward.

Definition 3.27:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in $P(F_A,E)$. Then the intersection of all closed IVFNS set containing (f_A,E) is called the closure of (f_A,E) and is denoted by cl (f_A,E) and defined by cl $(f_A,E)=\cap\{ (g_A,E)/ (g_A,E)$ is a IVFNS closed set containing $(f_A,E)\}$. Thus cl (f_A,E) is the smallest IVFNS closed set containing (f_A,E).

Example 3.28:

Let us consider an interval valued fuzzy neutrosophic soft topology $\tau=\{(\varphi_A, E), (F_A, E), (f_A^1,E), (f_A^2,E), (f_A^3,E) ,(f_A^4,E)\}$ as in example 3.2 and let
$(f_A, E) = \{e_1=\{<h_1 [0.1,0.2], [0.3,0.4], [0.5,0.6]>,$
$<h_2 [0,0.1], [0.4,0.5], [0.7,0.8]>, <h_3 [0,0], [0,0], [1,1]>\}$
$\{e_2 =\{<h_1 [0,0], [0.4,0.5], [0.6,0.7]>, <h_2 [0,0], [0,0], [1,1]>, <h_3 [1,1], [1,1], [0,0]>\}$
$\{e_3=\{<h_1 [1,1], [1,1], [0,0]>, <h_2 [1,1], [1,1], [0,0]>,$
$<h_3 [1,1], [1,1], [0,0]>\}$ be an IVFNS set.
Then $cl(f_A,E) =\cap\{ (g_A,E)/ (g_A,E)$ is a IVFNS closed set containing $(f_A,E)\} = (f_A^1,E)^c \cap(f_A^4,E)^c = (f_A^4,E)^c$
Since $(f_A,E)\subseteq (f_A^1,E)^c$ and $(f_A,E)\subseteq (f_A^4,E)^c$

Proposition 3.29:

For any two IVFNS sets (f_A,E) and (g_A,E) is an interval valued fuzzy neutrosophic soft topological space $((F_A, E), \tau)$ on $P(F_A, E)$ then

(i) cl (f_A,E) is the smallest IVFNS closed set containing (f_A,E).
(ii) (f_A,E) is IVFNS closed if and only if (f_A,E) = cl (f_A,E)
(iii) $(g_A,E)\subseteq (f_A,E)$ implies $cl(g_A,E)\subseteq cl(f_A,E)$.
(iv) $cl(cl(f_A,E)) = cl(f_A,E)$.
(v) $cl(\varphi_A,E) = (\varphi_A,E)$ and $cl(F_A,E) = (F_A,E)$.
(vi) $cl((g_A,E)\cup (f_A,E)) = cl(g_A,E)\cup cl(f_A,E)$.
(vii) $cl((g_A,E)\cap (f_A,E)) \subseteq cl(g_A,E)\cap cl(f_A,E)$.

Proof:

(i) and (ii) follows from the definition.

(iii) Since $(g_A,E)\subseteq (f_A,E)$ implies all the closed set containing (f_A,E) also contain (g_A,E). Therefore $\cap \{(g_A^*,E)/ (g_A^*,E)$ is an IVFNS closed set containing $(g_A,E)\}\subseteq\cap\{ (f_A^*,E)/ (f_A^*,E)$ is an IVFNS closed set containing $(f_A,E)\}$. So cl $(g_A,E)\subseteq$cl (f_A,E).

(iv) $cl(cl (f_A,E)) = \cap\{ (g_A,E)/ (g_A,E)$ is an IVFNS closed set containing cl $(f_A,E)\}$ and since cl (f_A,E) is the smallest closed IVFNS set containing cl (f_A,E). Therefore $cl(cl (f_A,E)) = cl (f_A,E)$.

(v) Proof is obvious.

(vi) Since $cl(g_A,E) \supseteq (g_A,E)$ and $cl(f_A,E)\supseteq (f_A,E)$, we have $cl(g_A,E)\cup cl (f_A,E)\supseteq (g_A,E)\cup (f_A,E)$. This implies $cl(g_A,E)\cup cl(f_A,E)\supseteq cl((g_A,E)\cup (f_A,E))$ -----(1). And since $(g_A,E)\cup (f_A,E)\supseteq (g_A,E)$ and $(g_A,E)\cup (f_A,E) \supseteq (f_A,E)$ so $cl((g_A,E)\cup(f_A,E))\supseteq cl(g_A,E)$ and $cl((g_A,E)\cup(f_A,E))\supseteq cl(f_A,E)$. Therefore $cl((g_A,E)\cup(f_A,E))\supseteq cl(g_A,E)\cup cl(f_A,E)$-----(2).
Form (1) and (2) $cl(g_A,E)\cup cl(f_A,E) = cl(g_A,E)\cup cl(f_A,E)$.

(vii) Since $(g_A,E) \supseteq (g_A,E)\cap (f_A,E)$ and $(f_A,E)\supseteq (g_A,E)\cap (f_A,E)$ so cl $(g_A,E)\supseteq cl((g_A,E)\cap(f_A,E))$ and $cl(f_A,E)\supseteq cl((g_A,E)\cap(f_A,E))$.
Hence $cl ((g_A,E)\cap cl(f_A,E)\supseteq cl((g_A,E)\cap(f_A,E))$.

Theorem 3.30:

Let $((F_A, E), \tau)$ be an interval valued fuzzy neutrosophic soft topological space on (F_A,E) and (f_A,E) be an IVFNS set in $P(F_A,E)$. Then the collection $\tau_{(f_A,E)} = \{(f_A,E)\cap(g_A,E)/(g_A,E)\in\tau\}$ is an interval valued fuzzy neutrosophic soft topology on the interval valued fuzzy neutrosophic soft set (f_A,E).

Proof:

(i) Since $(\varphi_A, E), (F_A, E) \in\tau$, $(f_A, E)= (f_A, E)\cap (F_A, E)$ and $(\varphi_A, E),= (f_A, E)\cap (\varphi_A, E)$. Therefore $(\varphi_A, E), (f_A, E) \in\tau$.

(ii) Let $\{(f_A^i,E)/i=1,2,3,....n\}$ be a finite family of IVFNS open sets in $\tau_{(f_A,E)}$, then for each i=1,2,3..n there exist $(g_A^i,E)\in\tau$ such that $(f_A^i,E) = (f_A, E)\cap (g_A^i,E)$ Now $\bigcap_{i=1}^{n} (f_A^i,E) =$

$\bigcap_{i=1}^{n} [(f_A,E)\cap(g_A^i,E)] = (f_A,E)\cap\left(\bigcap_{i=1}^{n}[g_A^i,E]\right)$ and since $\left(\bigcap_{i=1}^{n}[g_A^i,E]\right)\in\tau$ so $\bigcap_{i=1}^{n} (f_A^i,E)\in\tau_{(f_A,E)}$.

(iii) Let $\{(f_A^k,E)/k\in K\}$ be an arbitrary family of interval valued fuzzy neutrosophic soft open sets in $\tau_{(f_A,E)}$, then for each k∈K, there exist $(g_A^k,E)\in\tau$ such that (f_A^k,E)=$(f_A, E)\cap (g_A^k,E)$
Now $\bigcup_{k\in K}(f_A^k,E) = \bigcup_{k\in K}((f_A,E)\cap(g_A^k,E)) = (f_A,E)\cap\bigcup_{k\in K}(g_A^k,E)$ and since $\bigcup_{k\in K}(g_A^k,E)\in\tau$.

So $\bigcup_{k\in K}(f_A^k, E) \in \tau_{(f_A,E)}$.

Definition 3.31:

Let $((F_A, E), \tau)$ be an IVFNS topological space on (F_A, E) and (f_A, E) be an IVFNS set in $P(F_A, E)$. Then the IVFNS topology.

$\tau_{(f_A,E)} = \{(f_A, E) \cap (g_A, E) / (g_A, E) \in \tau\}$ is called interval valued fuzzy neutrosophic soft subspace topology (IVFNS subspace topology) and $((f_A, E), \tau_{(f_A,E)})$ is called interval valued fuzzy neutrosophic soft subspace of $((F_A, E), \tau)$.

Example 3.32:

Let us consider the interval valued fuzzy neutrosophic soft topology $\tau=\{(\varphi_A, E), (F_A, E), (f_A^1, E), (f_A^2, E), (f_A^3, E), (f_A^4, E)\}$ as in the example 3.2 and an IVFNS-set

$(f_A,E)=\{e_1=\{<h_1[0.2,0.3],[0.3,0.4],[0,0.1]>,$
$<h_2[0.5,0.6],[0.4,0.5],[0.1,0.2]>,$
$<h_3[0.2,0.3],[0.5,0.6],[0.6,0.7]>\}$
$\{e_2=\{<h_1[0.3,0.4],[0.5,0.6],[0.1,0.2]>,$
$<h_2[0.4,0.5],[0.6,0.7],[0.2,0.3]>,$
$<h_3[0.4,0.5],[0.4,0.5],[0.2,0.3]>\}$
$\{e_3=\{<h_1[0,0],[0,0],[1,1]>, <h_2[0,0],[0,0],[1,1]>,$
$<h_3[0,0],[0,0],[1,1]>\}$ be an IVFNS set

$(f_A, E)\cap (F_A, E) = (f_A, E)$

$(f_A, E)\cap (\varphi_A, E) = (\varphi_A, E)$

$(f_A, E)\cap (f_A^1, E) = (g_A^1, E) =$
$\{e_1=\{<h_1[0.2,0.3],[0.3,0.4],[0.2,0.3]>,$
$<h_2[0.4,0.5],[0.4,0.5],[0.1,0.2]>,$
$<h_3[0.2,0.3],[0.5,0.6],[0.6,0.7]>\}$
$\{e_2=\{<h_1[0.3,0.4],[0.5,0.6],[0.2,0.3]>,$
$<h_2[0.4,0.5],[0.6,0.7],[0.2,0.3]>, <h_3[0,0],[0,0],[1,1]>\}$
$\{e_3=\{<h_1[0,0],[0,0],[1,1]>, <h_2[0,0],[0,0],[1,1]>,$
$<h_3[0,0],[0,0],[1,1]>\}$

$(f_A, E)\cap (f_A^2, E) = (g_A^2, E) =$
$\{e_1=\{<h_1[0.2,0.3],[0.3,0.4],[0.1,0.2]>,$
$<h_2[0.5,0.6],[0.4,0.5],[0.2,0.3]>, <h_3[0.2,0.3],[0.5,0.6],[0.6,0.7]>\}$
$\{e_2=\{<h_1[0.2,0.3],[0.4,0.5],[0.1,0.2]>,$
$<h_2[0.4,0.5],[0.6,0.7],[0.2,0.3]>, <h_3[0,0],[0,0],[1,1]>\}$
$\{e_3=\{<h_1[0,0],[0,0],[1,1]>, <h_2[0,0],[0,0],[1,1]>,$
$<h_3[0,0],[0,0],[1,1]>\}$

$(f_A, E)\cap (f_A^3, E) = (g_A^3, E) =$
$\{e_1=\{<h_1[0.2,0.3],[0.3,0.4],[0.2,0.3]>,$
$<h_2[0.4,0.5],[0.4,0.5],[0.2,0.3]>,$
$<h_3[0.2,0.3],[0.5,0.6],[0.6,0.7]>\}$
$\{e_2=\{<h_1[0.2,0.3],[0.4,0.5],[0.2,0.3]>,$
$<h_2[0.4,0.5],[0.6,0.7],[0.2,0.3]>, <h_3[0,0],[0,0],[1,1]>\}$
$\{e_3=\{<h_1[0,0],[0,0],[1,1]>, <h_2[0,0],[0,0],[1,1]>,$
$<h_3[0,0],[0,0],[1,1]>\}$

$(f_A, E)\cap (f_A^4, E) = (g_A^4, E) =$
$\{e_1=\{<h_1[0.2,0.3],[0.3,0.4],[0.1,0.2]>,$
$<h_2[0.5,0.6],[0.4,0.5],[0.1,0.2]>,$
$<h_3[0.2,0.3],[0.5,0.6],[0.6,0.7]>\}$
$\{e_2=\{<h_1[0.3,0.4],[0.5,0.6],[0.1,0.2]>,$
$<h_2[0.4,0.5],[0.6,0.7],[0.2,0.3]>, <h_3[0,0],[0,0],[1,1]>\}$
$\{e_3=\{<h_1[0,0],[0,0],[1,1]>, <h_2[0,0],[0,0],[1,1]>,$
$<h_3[0,0],[0,0],[1,1]>\}$

Thus $\tau_{(f_A,E)} = \{(\varphi_A, E), (F_A, E), (g_A^1, E), (g_A^2, E), (g_A^3, E), (g_A^4, E)\}$ is an interval valued fuzzy neutrosophic soft subspace topology for τ and $((f_A, E), \tau_{(f_A,E)})$ is called interval valued fuzzy neutrosophic soft subspace of $((F_A, E), \tau)$.

Theorem 3.33:

Let $((\eta_A,E), \tau^1)$ be a IVFNS topological subspace of $((\xi_A, E),\tau^2)$ and let $((\xi_A, E),\tau^2)$ be a IVFNS topological subspace of $((F_A, E), \tau)$. Then $((\eta_A,E), \tau^1)$ is also an IVFNS topological subspace of $((F_A, E), \tau)$..

Proof:

Since $(\eta_A,E) \subseteq (\xi_A, E) \subseteq (F_A, E)$, $((\eta_A,E), \tau^1)$ is an interval valued fuzzy neutrosophic soft topological subspace of $((F_A, E), \tau)$. if and only if $\tau_{(\eta_A,E)} = \tau^1$. Let $(f_A^1, E) \in \tau^1$, now since $((\eta_A,E), \tau^1)$ is an IVFNS topological subspace of $((\xi_A, E),\tau^2)$ ie., $\tau^2{}_{(\eta_A,E)} = \tau^1$, so there exist $f_A^2, E) \in \tau^2$ such that $(f_A^1, E) = (\eta_A,E)\cap (f_A^2, E)$. But $((\xi_A, E),\tau^2)$ is an IVFNS topological subspace of $((F_A, E), \tau)$.Therefore there exist $(f_A, E) \in \tau$ such that $(f_A^2, E) = (\xi_A,E)\cap (f_A,E)$. Thus $(f_A^1, E) = (\eta_A,E)\cap (f_A^2, E) = (\eta_A,E)\cap (\xi_A, E)\cap (f_A,E) = (\eta_A,E)\cap (f_A,E)$. So $(f_A^1, E) \in \tau_{(\eta_A,E)}$ implies $\tau^1 \subseteq \tau_{(\eta_A,E)}$ ---(1).

Now assume, $(g_A, E) \in \tau_{(\eta_A,E)}$ ie., there exist $(h_A,E) \in \tau$ such that $(g_A,E) = (\eta_A,E)\cap (h_A,E)$. But $(\xi_A, E)\cap (h_A,E) \in \tau_{(\xi_A,E)} = \tau^2$.So $(\eta_A,E)\cap ((\xi_A, E)\cap (h_A,E)) = (\eta_A,E)\cap (h_A,E) = (g_A,E)$. We have

$(g_A,E) \in \tau^1$ implies $\tau_{(\eta_A,E)} \subseteq \tau^1$--------(2). From (1) and (2) $\tau^1 = \tau_{(\eta_A,E)}$. Hence the proof.

Theorem 3.34:

Let $((F_A, E), \tau)$.be an IVFNS topological space of (F_A, E). B be an basis for τ and (f_A, E) be an IVFNS set in $P(F_A, E)$. Then the family $B_{(f_A,E)} = \{(f_A, E) \cap (g_A, E) / (g_A, E) \in B\}$ is an IVFNS basis for subspace topology $\tau_{(f_A,E)}$.

Proof:

Let $(h_A,E) \in \tau_{(f_A,E)}$, then there exist an IVFNS set $(g_A, E) \in \tau$, such that $(h_A,E) = (f_A, E) \cap (g_A, E)$. Since B is a base for τ, there exist sub-collection $\{\{(\psi_A^i, E) / i \in I\}$ of B, such that $(g_A, E) = \bigcup_{i \in I} (\psi_A^i, E)$. Therefore $(h_A,E) = (f_A, E) \cap (g_A,E) = (f_A, E) \cap (\bigcup_{i \in I} (\psi_A^i, E)) = \bigcup_{i \in I} ((f_A, E) \cap (\psi_A^i, E))$. Since $(f_A, E) \cap (\psi_A^i, E) \in B_{(f_A,E)}$ implies $B_{(f_A,E)}$ is an IVFNS basis for the IVFNS subspace topology $\tau_{(f_A,E)}$.

4. Conclusion

In this paper the notion of topological space in interval valued fuzzy neutrosophic soft sets is introduced. Further, some of its operators and properties of topology in IVFNS set are established.

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Soft Neutrosophic Bi-LA-semigroup and Soft Neutrosophic N-LA-seigroup

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Abstract. Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic bi-LA-semigroup,soft neutosophic sub bi-LA-semigroup, soft neutrosophic N -LA-semigroup with the discuission of some of their characteristics. We also introduced a new type of soft neutrophic bi-LAsemigroup, the so called soft strong neutrosophic bi-LAsemigoup which is of pure neutrosophic character. This is also extend to soft neutrosophic strong N-LA-semigroup. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

Keywords: Neutrosophic bi-LA-semigroup,Neutrosophic N -LA-semigroup, Soft set, Soft neutrosophic bi-LAisemigroup. Soft Neutrosophic N -LA-semigroup.

1 Introduction

Florentine Smarandache for the first time introduced the concept of neutrosophy in 1995, which is basically a new branch of philosophy which actually studies the origin, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T , the percentage of indeterminacy in a subset I , and the percentage of falsity in a subset F . Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interval valued fuzzy set. Neutrosophic logic is used to overcome the problems of impreciseness, indeterminate, and inconsistencies of date etc. The theory of neutrosophy is so applicable to every field of algebra. W.B. Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings, neutrosophic vector spaces, neutrosophic groups, neutrosophic bigroups and neutrosophic N -groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrosophic N -semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N -loops, and so on. Mumtaz ali et. al. introduced nuetrosophic LA -semigroups . Soft neutrosophic LA-semigroup has been introduced by Florentin Smarandache et.al.

Molodtsov introduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation research, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in $[2,9,10]$. Some properties and algebra may be found in $[1]$. Feng et al. introduced soft semirings in $[5]$. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in $[6]$. Some other concepts together with fuzzy set and rough set were shown in $[7,8]$.

In this paper we introduced soft nuetrosophic bi-LA-semigroup and soft neutrosophic N -LA-semigroup and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic bi-LA-semigroup, soft neutrosophic strong bi-LA-semigroup, and some of their properties are discussed. In the last section soft neutrosophic N -LA-semigroup and their corresponding strong theory have been constructed with some of their properties.

2 Fundamental Concepts

Definition 1. Let $(BN(S),*,\circ)$ be a nonempty set with two binary operations $*$ and $\circ$. $(BN(S),*,\circ)$ is said to be a neutrosophic bi-LA-semigroup if $BN(S)=P_1\cup P_2$ where atleast one of $(P_1,*)$ or $(P_2,\circ)$ is a neutrosophic LA-semigroup and other is just an LA- semigroup. P_1 and P_2 are proper subsets of $BN(S)$.

If both $(P_1,*)$ and $(P_2,\circ)$ in the above definition are neutrosophic LA-semigroups then we call $(BN(S),*,\circ)$ a strong neutrosophic bi-LA-semigroup.

Definition 2. Let $(BN(S)=\mathrm{P}_1\cup P;:*,\circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T,\circ,*)$ is said to be a neutrosophic sub bi-LA-semigroup of $BN(S)$ if

1. $T=T_1\cup T_2$ where $T_1=P_1\cap T$ and $T_2=P_2\cap T$ and
2. At least one of $(T_1,\circ)$ or $(T_2,*)$ is a neutrosophic LA-semigroup.

Definition 3 . Let $(BN(S)=\mathrm{P}_1\cup P,*,\circ)$ be a neutrosophic bi-LA-semigroup. A proper subset $(T,\circ,*)$ is said to be a neutrosophic strong sub bi-LA-semigroup of $BN(S)$ if

1. $T=T_1\cup T_2$ where $T_1=P_1\cap T$ and $T_2=P_2\cap T$ and
2. $(T_1,\circ)$ and $(T_2,*)$ are neutrosophic strong LA-semigroups.

Definition 4. Let $(BN(S)=\mathrm{P}_1\cup P,*,\circ)$ be any neutrosophic bi-LA-semigroup. Let J be a proper subset of $BN(S)$ such that $J_1=J\cap P_1$ and $J_2=J\cap P_2$ are ideals of P_1 and P_2 respectively. Then J is called the neutrosophic biideal of $BN(S)$.

Definition 5. Let $(BN(S),*,\circ)$ be a strong neutrosophic bi-LA-semigroup where $BN(S)=\mathrm{P}_1\cup P_2$ with $(P_1,*)$ and $(P_2,\circ)$ be any two neutrosophic LA-semigroups. Let J be a proper subset of $BN(S)$ where $I=I_1\cup I_2$ with $I_1=I\cap P_1$ and $I_2=I\cap P_2$ are neutrosophic ideals of the neutrosophic LA-semigroups P_1 and P_2 respectively. Then I is called or defined as the strong neutrosophic biideal of $BN(S)$.

Definition 6. Let $\{S(N),*_1,...,*_2\}$ be a non-empty set with N -binary operations defined on it. We call $S(N)$ a neutrosophic N -LA-semigroup (N a positive integer) if the following conditions are satisfied.

1) $S(N)=S_1\cup ...S_N$ where each S_i is a proper subset of $S(N)$ i.e. $S_i\subset S_j$ or $S_j\subset S_i$ if $i\neq j$.
2) $(S_i,*_i)$ is either a neutrosophic LA-semigroup or an LA-semigroup for $i=1,2,3,...,N$.

If all the N -LA-semigroups $(S_i,*_i)$ are neutrosophic LA-semigroups (i.e. for $i=1,2,3,...,N$) then we call $S(N)$ to be a neutrosophic strong N -LA-semigroup.

Definition 7. Let $S(N)=\{S_1\cup S_2\cup ....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic N -LA-semigroup. A proper subset $P=\{\mathrm{P}_1\cup P_2\cup ....\mathrm{P}_N,*_1,*_2,...,*_N\}$ of $S(N)$ is said to be a neutrosophic sub N -LA-semigroup if $P_i=P\cap S_i, i=1,2,...,N$ are sub LA-semigroups of S_i in which atleast some of the sub LA-semigroups are neutrosophic sub LA-semigroups.

Definition 8. Let $S(N)=\{S_1\cup S_2\cup ....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic strong N -LA-semigroup. A proper subset $T=\{\mathrm{T}_1\cup T_2\cup ....\cup \mathrm{T}_N,*_1,*_2,...,*_N\}$ of $S(N)$ is said to be a neutrosophic strong sub N -LA-semigroup if each $(T_i,*_i)$ is a neutrosophic sub LA-semigroup of $(S_i,*_i)$ for $i=1,2,...,N$ where $T_i=S_i\cap T$.

Definition 9. Let $S(N)=\{S_1\cup S_2\cup ....S_N,*_1,*_2,...,*_N\}$ be a neutrosophic N -LA-semigroup. A proper subset $P=\{\mathrm{P}_1\cup P_2\cup ....\cup P_N,*_1,*_2,...,*_N\}$ of $S(N)$ is said to be a neutrosophic N -ideal, if the following conditions are true,

1. P is a neutrosophic sub N -LA-semigroup of

$S(N)$.

2. Each $P_i = S \cap P_i, i = 1,2,...,N$ is an ideal of S_i.

Definition 10. Let $S(N) = \{S_1 \cup S_2 \cup ....S_N, *_1, *_2, ..., *_N\}$ be a neutrosophic strong N-LA-semigroup. A proper subset $J = \{J_1 \cup J_2 \cup ....J_N, *_1, *_2, ..., *_N\}$ where $J_t = J \cap S_t$ for $t = 1,2,...,N$ is said to be a neutrosophic strong N-ideal of $S(N)$ if the following conditions are satisfied.

1) Each it is a neutrosophic sub LA-semigroup of $S_t, t = 1,2,...,N$ i.e. It is a neutrosophic strong N-sub LA-semigroup of $S(N)$.
2) Each it is a two sided ideal of S_t for $t = 1,2,...,N$.

Similarly one can define neutrosophic strong N-left ideal or neutrosophic strong right ideal of $S(N)$.

A neutrosophic strong N-ideal is one which is both a neutrosophic strong N-left ideal and N-right ideal of $S(N)$.

Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 11. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(a)$ may be considered as the set of a-elements of the soft set (F, A), or as the set of a-approximate elements of the soft set.

Example 1. Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or sentence. Let

$$E = \left\{ \begin{matrix} \text{high rent,normal rent,} \\ \text{in good condition,in bad condition} \end{matrix} \right\}.$$

Let us consider a soft set (F, A) which describes the attractiveness of shops that Mr. Z is taking on rent. Suppose that there are five houses in the universe $U = \{s_1, s_2, s_3, s_4, s_5\}$ under consideration, and that $A = \{a_1, a_2, a_3\}$ be the set of parameters where

a_1 stands for the parameter 'high rent,

a_2 stands for the parameter 'normal rent,

a_3 stands for the parameter 'in good condition.

Suppose that

$$F(a_1) = \{s_1, s_4\},$$
$$F(a_2) = \{s_2, s_5\},$$
$$F(a_3) = \{s_3\}.$$

The soft set (F, A) is an approximated family $\{F(a_i), i = 1,2,3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Then (F, A) is a soft set as a collection of approximations over U, where

$$F(a_1) = high\ rent = \{s_1, s_2\},$$
$$F(a_2) = normal\ rent = \{s_2, s_5\},$$
$$F(a_3) = in\ good\ condition = \{s_3\}.$$

Definition 12. For two soft sets (F, A) and (H, B) over U, (F, A) is called a soft subset of (H, B) if

1. $A \subseteq B$ and
2. $F(a) \subseteq H(a)$, for all $x \in A$.

This relationship is denoted by $(F, A) \subset (H, B)$. Similarly (F, A) is called a soft superset of (H, B) if (H, B) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, B)$.

Definition 13. Two soft sets (F, A) and (H, B) over U are called soft equal if (F, A) is a soft subset of (H, B) and (H, B) is a soft subset of (F, A).

Definition 14. Let (F, A) and (K, B) be two soft sets over a common universe U such that $A \cap B \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (K, B) = (H, C)$ where (H, C) is defined as $H(c) = F(c) \cap K(c)$ for all $c \in C = A \cap B$.

Definition 15. The extended intersection of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if c} \in A - B, \\ G(c) & \text{if c} \in B - A, \\ F(c) \cap G(c) & \text{if c} \in A \cap B. \end{cases}$$

We write $(F,A) \cap_{\varepsilon} (K,B) = (H,C)$.

Definition 16. The restricted union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as $H(c) = F(c) \cup G(c)$ for all $c \in C$. We write it as $(F,A) \cup_R (K,B) = (H,C)$.

Definition 17. The extended union of two soft sets (F,A) and (K,B) over a common universe U is the soft set (H,C), where $C = A \cup B$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if c} \in A - B, \\ G(c) & \text{if c} \in B - A, \\ F(c) \cup G(c) & \text{if c} \in A \cap B. \end{cases}$$

We write $(F,A) \cup_{\varepsilon} (K,B) = (H,C)$.

3 Soft Neutrosophic Bi-LA-semigroup

Definition 18. Let $BN(S)$ be a neutrosophic bi-LA-semigroup and (F,A) be a soft set over $BN(S)$. Then (F,A) is called soft neutrosophic bi-LA-semigroup if and only if $F(a)$ is a neutrosophic sub bi-LA-semigroup of $BN(S)$ for all $a \in A$.

Example 2. Let $BN(S) = \{\langle S_1 \cup I \rangle \cup \langle S_2 \cup I \rangle\}$ be a neutrosophic bi-LA-semigroup where $\langle S_1 \cup I \rangle = \{1,2,3,4,1I,2I,3I,4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

$\langle S_2 \cup I \rangle = \{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then clearly (F,A) is a soft neutrosophic bi-LA-semigroup over $BN(S)$, where

$$F(a_1) = \{1,1I\} \cup \{2,3,3I\},$$
$$F(a_2) = \{2,2I\} \cup \{1,3,1I,3I\},$$
$$F(a_3) = \{4,4I\} \cup \{1I,3I\}.$$

Proposition 1. Let (F,A) and (K,D) be two soft neutrosophic bi-LA-semigroups over $BN(S)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{D})$ is soft neutrosophic bi-LA-semigroup over $BN(S)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{D})$ is soft neutrosophic bi-LA-semigroup over $BN(S)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{D})$ is soft neutrosophic bi-LA-semigroup over $BN(S)$.

Remark 1. Let (F,A) and (K,D) be two soft neutrosophic bi-LA-semigroups over $BN(S)$. Then

1. Their extended union $(F,A)\cup_E(K,\mathrm{D})$ is not soft neutrosophic bi-LA-semigroup over $BN(S)$.
2. Their restricted union $(F,A)\cup_R(K,\mathrm{D})$ is not soft neutrosophic bi-LA-semigroup over $BN(S)$.
3. Their OR operation $(F,A)\vee(K,\mathrm{D})$ is not soft neutrosophic bi-LA-semigroup over $BN(S)$.

One can easily proved $(1),(2),$ and (3) by the help of examples.

Definition 19. Let (F,A) and (K,D) be two soft neutrosophic bi-LA-semigroups over $BN(S)$. Then (K,D) is called soft neutrosophic sub bi-LA-semigroup of (F,A), if

1. $D\subseteq A$.
2. $\mathrm{K}(a)$ is a neutrosophic sub bi-LA-semigroup of $F(a)$ for all $a\in A$.

Example 3. Let (F,A) be a soft neutrosophic bi-LA-semigroup over $BN(S)$ in Example (1). Then clearly (K,D) is a soft neutrosophic sub bi-LA-semigroup of (F,A) over $BN(S)$, where

$$K(a_1)=\{1,1I\}\cup\{3,3I\},$$
$$K(a_2)=\{2,2I\}\cup\{1,1I\}.$$

Theorem 1. Let (F,A) be a soft neutrosophic bi-LA-semigroup over $BN(S)$ and $\{(H_j,B_j): j\in J\}$ be a non-empty family of soft neutrosophic sub bi-LA-semigroups of (F,A). Then

1) $\bigcap_{j\in J}R\,(H_j,B_j)$ is a soft neutrosophic sub bi-LA-semigroup of (F,A).

2) $\wedge_{j\in J}R\,(H_j,B_j)$ is a soft neutrosophic sub bi-LA-semigroup of (F,A).

3) $\bigcup_{j\in J}\varepsilon\,(H_j,B_j)$ is a soft neutrosophic sub bi-LA-semigroup of (F,A) if $B_j\cap B_k=\phi$ for all $j,k\in J$.

Definition 20. Let (F,A) be a soft set over a neutrosophic bi-LA-semigroup $BN(S)$. Then (F,A) is called soft neutrosophic biideal over $BN(S)$ if and only if $F(a)$ is a neutrosophic biideal of $BN(S)$, for all $a\in A$.

Example 4. Let $BN(S)=\{\langle S_1\cup I\rangle\cup\langle S_2\cup I\rangle\}$ be a neutrosophic bi-LA-semigroup, where $\langle S_1\cup I\rangle=\{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

And $\langle S_2 \cup I \rangle = \{1,2,3,I,2I,3I\}$ be another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic biideal over $BN(S)$, where

$$F(a_1)=\{1,1I,3,3I\}\cup\{2,2I\},$$
$$F(a_2)=\{1,3,1I,3I\}\cup\{2,3,2I,3I\}.$$

Proposition 2. Every soft neutrosophic biideal over a neutrosophic bi-LA-semigroup is trivially a soft neutrosophic bi-LA-semigroup but the conver is not true in general.

One can easily see the converse by the help of example.

Proposition 3 . Let (F,A) and (K,D) be two soft neutrosophic biideals over $BN(S)$. Then

1) Their restricted union $(F,A)\cup_R(K,D)$ is not a soft neutrosophic biideal over $BN(S)$.

2) Their restricted intersection $(F,A)\cap_R(K,D)$ is a soft neutrosophic biideal over $BN(S)$.

3) Their extended union $(F,A)\cup_\varepsilon(K,D)$ is not a soft neutrosophic biideal over $BN(S)$.

4) Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ is a soft neutrosophic biideal over $BN(S)$.

Proposition 4. Let (F,A) and (K,D) be two soft neutrosophic biideals over $BN(S)$. Then

1. Their OR operation $(F,A)\vee(K,D)$ is not a soft neutrosophic biideal over $BN(S)$.

2. Their AND operation $(F,A)\wedge(K,D)$ is a soft neutrosophic biideal over $BN(S)$.

Definition 21 . Let (F,A) and (K,D) be two soft neutrosophic bi- LA-semigroups over $BN(S)$. Then (K,D) is called soft neutrosophic biideal of (F,A), if

1) $B\subseteq A$, and

2) $K(a)$ is a neutrosophic biideal of $F(a)$, for all $a\in A$.

Example 5. Let (F,A) be a soft neutrosophic bi-LA-semigroup over $BN(S)$ in Example $(*)$. Then (K,D) is a soft neutrosophic biideal of (F,A) over $BN(S)$, where

$$K(a_1)=\{1\mathrm{I},3I\}\cup\{2,2I\},$$
$$K(a_2)=\{1,3,1I,3I\}\cup\{2\mathrm{I},3\mathrm{I}\}.$$

Theorem 2. A soft neutrosophic biideal of a soft neutrosophic bi-LA-semigroup over a neutrosophic bi-LA_semigroup is trivially a soft neutosophic sub bi-LA-semigroup but the converse is not true in general.

Proposition 5. If (F',A') and (G',B') are soft neutrosophic biideals of soft neutrosophic bi-LA-semigroups (F,A) and (G,B) over neutrosophic bi-LA-semigroups $N(S)$ and $N(T)$ respectively. Then $(F',A')\times(G',B')$ is a soft neutrosophic biideal of soft neutrosophic bi-LA-semigroup $(F,A)\times(G,B)$ over $N(S)\times N(T)$.

Theorem 3. Let (F,A) be a soft neutrosophic bi-LA-semigroup over $BN(S)$ and $\{(H_j,B_j): j\in J\}$ be a non-empty family of soft neutrosophic biideals of (F,A). Then

1) $\bigcap_{\substack{R \\ j\in J}}\left(H_j, B_j\right)$ is a soft neutrosophicbi ideal of (F, A).

2) $\bigwedge_{j\in J}\left(H_j, B_j\right)$ is a soft neutrosophic biideal of (F, A).

3) $\bigcup_{\substack{\mathcal{E} \\ j\in J}}\left(H_j, B_j\right)$ is a soft neutrosophic biideal of (F, A).

4) $\bigvee_{j\in J}\left(H_j, B_j\right)$ is a soft neutrosophic biideal of (F, A).

4 Soft Neutrosophic Storng Bi-LA-semigroup

Definition 22. Let $BN(S)$ be a neutrosophic bi-LA-semigroup and (F, A) be a soft set over $BN(S)$. Then (F, A) is called soft neutrosophic strong bi-LA-semigroup if and only if $F(a)$ is a neutrosophic strong sub bi-LA-semigroup for all $a \in A$.

Example 6. Let $BN(S)$ be a neutrosophic bi-LA-semigroup in Example (1). Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is a soft neutrosophic strong bi-LA-semigroup over $BN(S)$,where

$$F(a_1) = \{1I, 2I, 3I, 4I\} \cup \{2I, 3I\},$$

$$F(a_2) = \{1I, 2I, 3I, 4I\} \cup \{1I, 3I\}.$$

Proposition 6. Let (F, A) and (K, D) be two soft neutrosophic strong bi-LA-semigroups over $BN(S)$. Then

1. Their extended intersection $(F, A) \cap_E (K, \mathrm{D})$ is soft neutrosophic strong bi-LA-semigroup over $BN(S)$.
2. Their restricted intersection $(F, A) \cap_R (K, \mathrm{D})$ is soft neutrosophic strong bi-LA-semigroup over $BN(S)$.
3. Their AND operation $(F, A) \wedge (K, \mathrm{D})$ is soft neutrosophic strong bi-LA-semigroup over $BN(S)$.

Remark 2. Let (F, A) and (K, D) be two soft neutrosophic strong bi-LA-semigroups over $BN(S)$. Then

1. Their extended union $(F, A) \cup_E (K, \mathrm{D})$ is not soft neutrosophic strong bi-LA-semigroup over $BN(S)$.
2. Their restricted union $(F, A) \cup_R (K, \mathrm{D})$ is not soft neutrosophic strong bi-LA-semigroup over $BN(S)$.
3. Their OR operation $(F, A) \vee (K, \mathrm{D})$ is not soft neutrosophic strong bi-LA-semigroup over $BN(S)$.

One can easily proved $(1), (2),$ and (3) by the help of examples.

Definition 23. Let (F, A) and (K, D) be two soft neutrosophic strong bi-LA-semigroups over $BN(S)$. Then (K, D) is called soft neutrosophic strong sub bi-LA-semigroup of (F, A), if

1. $B \subseteq A$.
2. $\mathrm{K}(a)$ is a neutrosophic strong sub bi-LA-semigroup of $F(a)$ for all $a \in A$.

Theorem 4. Let $\left(F, A\right)$ be a soft neutrosophic strong bi-LA-semigroup over $BN(S)$ and $\left\{\left(H_j, B_j\right) : j \in J\right\}$ be a non-empty family of soft neutrosophic strong sub bi-LA-semigroups of $\left(F, A\right)$. Then

1. $\bigcap_{\substack{R \\ j\in J}}\left(H_j, B_j\right)$ is a soft neutrosophic strong sub bi-LA-semigroup of $\left(F, A\right)$.
2. $\bigwedge_{\substack{R \\ j\in J}}\left(H_j, B_j\right)$ is a soft neutrosophic strong sub bi-LA-semigroup of $\left(F, A\right)$.
3. $\bigcup_{\substack{E \\ j\in J}}\left(H_j, B_j\right)$ is a soft neutrosophic strong

sub bi-LA-semigroup of (F,A) if
$B_j \cap B_k = \phi$ for all $j,k \in J$.

Definition 24. Let (F,A) be a soft set over a neutrosophic bi-LA-semigroup $BN(S)$. Then (F,A) is called soft neutrosophic strong biideal over $BN(S)$ if and only if $F(a)$ is a neutrosophic strong biideal of $BN(S)$, for all $a \in A$.

Example 7. Let $BN(S)$ be a neutrosophic bi-LA-semigroup in Example $(*)$. Let $A=\{a_1,a_2\}$ be a set of parameters. Then clearly (F,A) is a soft neutrosophic strong biideal over $BN(S)$, where

$$F(a_1)=\{1\mathrm{I},3I\}\cup\{1\mathrm{I},2I,3I\},$$
$$F(a_2)=\{1I,3I\}\cup\{2\mathrm{I},3\mathrm{I}\}.$$

Theorem 5. Every soft neutrosophic strong biideal over $BN(S)$ is a soft neutrosophic biideal but the converse is not true.

We can easily see the converse by the help of example.

Proposition 7. Every soft neutrosophic strong biideal over a neutrosophic bi-LA-semigroup is trivially a soft neutrosophic strong bi-LA-semigroup but the converse is not true in general.

Proposition 8. Every soft neutrosophic strong biideal over a neutrosophic bi-LA-semigroup is trivially a soft neutrosophic bi-LA-semigroup but the converse is not true in general.
One can easily see the converse by the help of example.

Proposition 9 . Let (F,A) and (K,D) be two soft neutrosophic strong biideals over $BN(S)$. Then

1. Their restricted union $(F,A)\cup_R (K,D)$ is not a soft neutrosophic strong biideal over $BN(S)$.
2. Their restricted intersection $(F,A)\cap_R (K,D)$ is a soft neutrosophic strong biideal over $BN(S)$.
3. Their extended union $(F,A)\cup_\varepsilon (K,D)$ is not a soft neutrosophic strong biideal over $BN(S)$.
4. Their extended intersection $(F,A)\cap_\varepsilon (K,D)$ is a soft neutrosophic strong biideal over $BN(S)$.
5. Their OR operation $(F,A)\vee(K,D)$ is not a soft neutrosophic biideal over $BN(S)$.
6. Their AND operation $(F,A)\wedge(K,D)$ is a soft neutrosophic biideal over $BN(S)$.

Definition 25. Let (F,A) and (K,D) be two soft neutrosophic strong bi- LA-semigroups over $BN(S)$. Then (K,D) is called soft neutrosophic strong biideal of (F,A), if

1. $D \subseteq A$, and
2. $K(a)$ is a neutrosophic strong biideal of $F(a)$, for all $a \in A$.

Theorem 6. A soft neutrosophic strong biideal of a soft neutrosophic strong bi-LA-semigroup over a neutrosophic bi-LA_semigroup is trivially a soft neutosophic strong sub bi-LA-semigroup but the converse is not true in general.

Proposition 10. If (F',A') and (G',B') are soft neutrosophic strong biideals of soft neutrosophic bi-LA-semigroups (F,A) and (G,B) over neutrosophic bi-LA-semigroups $N(S)$ and $N(T)$ respectively. Then $(F',A')\times(G',B')$ is a soft neutrosophic strong biideal of soft neutrosophic bi-LA-semigroup $(F,A)\times(G,B)$ over $N(S)\times N(T)$.

Theorem 7. Let (F,A) be a soft neutrosophic strong bi-LA-semigroup over $BN(S)$ and $\{(H_j,B_j): j\in J\}$ be a non-empty family of soft neutrosophic strong biideals of (F,A). Then

1. $\bigcap_{j\in J} R(H_j,B_j)$ is a soft neutrosophic strong bi

ideal of (F,A).

2. $\underset{j\in J}{\wedge}(H_j,B_j)$ is a soft neutrosophic strong biideal of (F,A).

3. $\underset{j\in J}{\cup_{\varepsilon}}(H_j,B_j)$ is a soft neutrosophic strong biideal of (F,A).

4. $\underset{j\in J}{\vee}(H_j,B_j)$ is a soft neutrosophic strong biideal of (F,A).

5 Soft Neutrosophic N-LA-semigroup

Definition 26. Let $\{S(N),*_1,*_2,...,*_N\}$ be a neutrosophic N-LA-semigroup and (F,A) be a soft set over $S(N)$.

Then (F,A) is called soft neutrosophic N-LA-semigroup if and only if $F(a)$ is a neutrosophic sub N-LA-semigroup of $S(N)$ for all $a\in A$.

Example 8. Let $\mathrm{S(N)}=\{\mathrm{S}_1\cup\mathrm{S}_2\cup\mathrm{S}_3,*_1,*_2,*_3\}$ be a neutrosophic 3-LA-semigroup where $S_1=\{1,2,3,4,1I,2I,3I,4I\}$ is a neutrosophic LA-semigroup with the following table.

*	1	2	3	4	1I	2I	3I	4I
1	1	4	2	3	1I	4I	2I	3I
2	3	2	4	1	3I	2I	4I	1I
3	4	1	3	2	4I	1I	3I	2I
4	2	3	1	4	2I	3I	1I	4I
1I	1I	4I	2I	3I	1I	4I	2I	3I
2I	3I	2I	4I	1I	3I	2I	4I	1I
3I	4I	1I	3I	2I	4I	1I	3I	2I
4I	2I	3I	1I	4I	2I	3I	1I	4I

$S_2=\{1,2,3,1I,2I,3I\}$ be another neutrosophic bi-LA-semigroup with the following table.

*	1	2	3	1I	2I	3I
1	3	3	3	3I	3I	3I
2	3	3	3	3I	3I	3I
3	1	3	3	1I	3I	3I
1I	3I	3I	3I	3I	3I	3I
2I	3I	3I	3I	3I	3I	3I
3I	1I	3I	3I	1I	3I	3I

And $S_3=\{1,2,3,I,2I,3I\}$ is another neutrosophic LA-semigroup with the following table.

.	1	2	3	I	2I	3I
1	3	3	2	3I	3I	2I
2	2	2	2	2I	2I	2I
3	2	2	2	2I	2I	2I
I	3I	3I	2I	3I	3I	2I
2I	2I	2I	2I	2I	2I	2I
3I	2I	2I	2I	2I	2I	2I

Let $A=\{a_1,a_2,a_3\}$ be a set of parameters. Then clearly (F,A) is a soft neutrosophic 3-LA-semigroup over $S(N)$, where

$$F(a_1)=\{1,1I\}\cup\{2,3,3I\}\cup\{2,2\,\mathrm{I}\},$$

$$F(a_2)=\{2,2I\}\cup\{1,3,1I,3I\}\cup\{2,3,2\,\mathrm{I},3\mathrm{I}\},$$

$$F(a_3)=\{4,4I\}\cup\{1I,3I\}\cup\{2\,\mathrm{I},3\mathrm{I}\}.$$

Proposition 11. Let (F,A) and (K,D) be two soft neutrosophic N-LA-semigroups over $S(N)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{D})$ is soft neutrosophic N-LA-semigroup over $S(N)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{D})$ is soft neutrosophic N-LA-semigroup over $S(N)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{D})$ is soft neutrosophic N-LA-semigroup over $S(N)$.

Remark 3. Let (F,A) and (K,D) be two soft neutrosophic N-LA-semigroups over $S(N)$. Then

1. Their extended union $(F,A)\cup_E(K,\mathrm{D})$ is not soft neutrosophic N-LA-semigroup over $S(N)$.
2. Their restricted union $(F,A)\cup_R(K,\mathrm{D})$ is not soft neutrosophic N-LA-semigroup over $S(N)$.
3. Their OR operation $(F,A)\vee(K,\mathrm{D})$ is not soft neutrosophic N-LA-semigroup over $S(N)$.

One can easily proved $(1),(2),$ and (3) by the help of examples.

Definition 27. Let (F,A) and (K,D) be two soft neutrosophic N-LA-semigroups over $S(N)$. Then (K,D) is called soft neutrosophic sub N-LA-semigroup of (F,A), if

1. $D\subseteq A$.
2. $\mathrm{K}(a)$ is a neutrosophic sub N-LA-semigroup of $F(a)$ for all $a\in A$.

Theorem 8. Let (F,A) be a soft neutrosophic N-LA-semigroup over $S(N)$ and $\{(H_j,B_j): j\in J\}$ be a non-empty family of soft neutrosophic sub N-LA-semigroups of (F,A). Then

1. $\bigcap_{j\in J}{}_R(H_j,B_j)$ is a soft neutrosophic sub N-LA-semigroup of (F,A).
2. $\wedge_{j\in J}{}_R(H_j,B_j)$ is a soft neutrosophic sub N-LA-semigroup of (F,A).
3. $\bigcup_{j\in J}{}_\varepsilon(H_j,B_j)$ is a soft neutrosophic sub N-LA-semigroup of (F,A) if $B_j\cap B_k=\phi$ for all $j,k\in J$.

Definition 28. Let (F,A) be a soft set over a neutrosophic N-LA-semigroup $S(N)$. Then (F,A) is called soft neutrosophic N-ideal over $S(N)$ if and only if $F(a)$ is a neutrosophic N-ideal of $S(N)$ for all $a\in A$.

Example 9. Consider Example (***).Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is a soft neutrosophic 3-ideal over $S(N)$, where

$$F(a_1)=\{1,1I\}\cup\{3,3I\}\cup\{2,2I\},$$

$$F(a_2)=\{2,2I\}\cup\{1I,3I\}\cup\{2,3,3\mathrm{I}\}.$$

Proposition 12. Every soft neutrosophic N-ideal over a neutrosophic N-LA-semigroup is trivially a soft neutrosophic N-LA-semigroup but the converse is not true in general.

One can easily see the converse by the help of example.

Proposition 13. Let (F,A) and (K,D) be two soft neutrosophic N-ideals over $S(N)$. Then

1. Their restricted union $(F,A)\cup_R(K,D)$ is not a soft neutrosophic N-ideal over $S(N)$.
2. Their restricted intersection $(F,A)\cap_R(K,D)$ is a soft neutrosophic N-ideal over $S(N)$.
3. Their extended union $(F,A)\cup_\varepsilon(K,D)$ is not a soft neutrosophic N-ideal over $S(N)$.

4. Their extended intersection $(F,A)\cap_{\varepsilon}(K,D)$ is a soft neutrosophic N-ideal over $S(N)$.

Proposition 15. Let (F,A) and (K,D) be two soft neutrosophic N-ideals over $S(N)$. Then

1. Their OR operation $(F,A)\vee(K,D)$ is a not soft neutrosophic N-ideal over $S(N)$.
2. Their AND operation $(F,A)\wedge(K,D)$ is a soft neutrosophic N-ideal over $S(N)$.

Definition 29 . Let (F,A) and (K,D) be two soft neutrosophic N- LA-semigroups over $S(N)$. Then (K,D) is called soft neutrosophic N-ideal of (F,A), if

1. $B\subseteq A$, and
2. $K(a)$ is a neutrosophic N-ideal of $F(a)$ for all $a\in A$.

Theorem 8. A soft neutrosophic N-ideal of a soft neutrosophic N-LA-semigroup over a neutrosophic N-LA-semigroup is trivially a soft neutosophic sub N-LA-semigroup but the converse is not true in general.

Proposition 16. If (F',A') and (G',B') are soft neutrosophic N-ideals of soft neutrosophic N-LA-semigroups (F,A) and (G,B) over neutrosophic N-LA-semigroups $N(S)$ and $N(T)$ respectively. Then $(F',A')\times(G',B')$ is a soft neutrosophic N-ideal of soft neutrosophic N-LA-semigroup $(F,A)\times(G,B)$ over $N(S)\times N(T)$.

Theorem 9. Let (F,A) be a soft neutrosophic N-LA-semigroup over $S(N)$ and $\{(H_j,B_j): j\in J\}$ be a non-empty family of soft neutrosophic N-ideals of (F,A). Then

1. $\cap_{R\,j\in J}(H_j,B_j)$ is a soft neutrosophic N-ideal of (F,A).
2. $\wedge_{j\in J}(H_j,B_j)$ is a soft neutrosophic N-ideal of (F,A).
3. $\cup_{\varepsilon\,j\in J}(H_j,B_j)$ is a soft neutrosophic N-ideal of (F,A).
4. $\vee_{j\in J}(H_j,B_j)$ is a soft neutrosophic N-ideal of (F,A).

6 Soft Neutrosophic Strong N-LA-semigroup

Definition 30. Let $\{S(N),*_1,*_2,...,*_N\}$ be a neutrosophic N-LA-semigroup and (F,A) be a soft set over $S(N)$. Then (F,A) is called soft neutrosophic strong N-LA-semigroup if and only if $F(a)$ is a neutrosophic strong sub N-LA-semigroup of $S(N)$ for all $a\in A$.

Example 10. Let $\mathrm{S(N)}=\{\mathrm{S}_1\cup\mathrm{S}_2\cup\mathrm{S}_3,*_1,*_2,*_3\}$ be a neutrosophic 3-LA-semigroup in Example 8. Let $A=\{a_1,a_2,a_3\}$ be a set of parameters. Then clearly (F,A) is a soft neutrosophic strong 3-LA-semigroup over $S(N)$, where

$$F(a_1)=\{1I\}\cup\{2I,3I\}\cup\{2\mathrm{I}\},$$

$$F(a_2)=\{2I\}\cup\{1I,3I\}\cup\{2\mathrm{I},3\mathrm{I}\},$$

Theorem 10. If $S(N)$ is a neutrosophic strong N-LA-semigroup, then (F,A) is also a soft neutrosophic strong N-LA-semigroup over $S(N)$.

Proposition 17. Let (F,A) and (K,D) be two soft neutrosophic strong N-LA-semigroups over $S(N)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{D})$ is soft neutrosophic strong N-LA-semigroup over $S(N)$.

2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{D})$ is soft neutrosophic strong N-LA-semigroup over $S(N)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{D})$ is soft neutrosophic strong N-LA-semigroup over $S(N)$.

Remark 4. Let (F,A) and (K,D) be two soft neutrosophic strong N-LA-semigroups over $S(N)$. Then

1. Their extended union $(F,A)\cup_E(K,\mathrm{D})$ is not soft neutrosophic strong N-LA-semigroup over $S(N)$.
2. Their restricted union $(F,A)\cup_R(K,\mathrm{D})$ is not soft neutrosophic strong N-LA-semigroup over $S(N)$.
3. Their OR operation $(F,A)\vee(K,\mathrm{D})$ is not soft neutrosophic strong N-LA-semigroup over $S(N)$.

One can easily proved $(1),(2),$ and (3) by the help of examples.

Definition 31. Let (F,A) and (K,D) be two soft neutrosophic strong N-LA-semigroups over $S(N)$. Then (K,D) is called soft neutrosophic strong sub N-LA-semigroup of (F,A), if

3. $D\subseteq A$.
4. $\mathrm{K}(a)$ is a neutrosophic strong sub N-LA-semigroup of $F(a)$ for all $a\in A$.

Theorem 11. Let (F,A) be a soft neutrosophic strong N-LA-semigroup over $S(N)$ and $\{(H_j,B_j): j\in J\}$ be a non-empty family of soft neutrosophic strong sub N-LA-semigroups of (F,A). Then

1. $\bigcap_{j\in J R}(H_j,B_j)$ is a soft neutrosophic strong sub N-LA-semigroup of (F,A).
2. $\wedge_{j\in J R}(H_j,B_j)$ is a soft neutrosophic strong sub N-LA-semigroup of (F,A).
3. $\bigcup_{j\in J\varepsilon}(H_j,B_j)$ is a soft neutrosophic strong sub N-LA-semigroup of (F,A) if $B_j\cap B_k=\phi$ for all $j,k\in J$.

Definition 32. Let (F,A) be a soft set over a neutrosophic N-LA-semigroup $S(N)$. Then (F,A) is called soft neutrosophic strong N-ideal over $S(N)$ if and only if $F(a)$ is a neutrosophic strong N-ideal of $S(N)$ for all $a\in A$.

Proposition 18. Every soft neutrosophic strong N-ideal over a neutrosophic N-LA-semigroup is trivially a soft neutrosophic strong N-LA-semigroup but the converse is not true in general.
One can easily see the converse by the help of example.

Proposition 19. Let (F,A) and (K,D) be two soft neutrosophic strong N-ideals over $S(N)$. Then

1. Their restricted union $(F,A)\cup_R(K,D)$ is not a soft neutrosophic strong N-ideal over $S(N)$.
2. Their restricted intersection $(F,A)\cap_R(K,D)$ is a soft neutrosophic N-ideal over $S(N)$.
3. Their extended union $(F,A)\cup_\varepsilon(K,D)$ is also a not soft neutrosophic strong N-ideal over $S(N)$.
4. Their extended intersection $(F,A)\cap_\varepsilon(K,D)$ is a soft neutrosophic strong N-ideal over $S(N)$.
5. Their OR operation $(F,A)\vee(K,D)$ is a not soft neutrosophic strong N-ideal over $S(N)$.
6. Their AND operation $(F,A)\wedge(K,D)$ is a

soft neutrosophic strong N-ideal over $S(N)$.

Definition 33 . Let (F,A) and (K,D) be two soft neutrosophic strong N- LA-semigroups over $S(N)$. Then (K,D) is called soft neutrosophic strong N-ideal of (F,A), if

1. $B \subseteq A$, and
2. $K(a)$ is a neutrosophic strong N-ideal of $F(a)$ for all $a \in A$.

Theorem 12. A soft neutrosophic strong N-ideal of a soft neutrosophic strong N-LA-semigroup over a neutrosophic N-LA_semigroup is trivially a soft neutosophic strong sub N-LA-semigroup but the converse is not true in general.

Theorem 13. A soft neutrosophic strong N-ideal of a soft neutrosophic strong N-LA-semigroup over a neutrosophic N-LA_semigroup is trivially a soft neutosophic strong N-ideal but the converse is not true in general.

Proposition 20. If $(F^{'},A^{'})$ and $(G^{'},B^{'})$ are soft neutrosophic strong N-ideals of soft neutrosophic strong N-LA-semigroups (F,A) and (G,B) over neutrosophic N-LA-semigroups $N(S)$ and $N(T)$ respectively. Then $(F^{'},A^{'})\times(G^{'},B^{'})$ is a soft neutrosophic strong N-ideal of soft neutrosophic strong N-LA-semigroup $(F,A)\times(G,B)$ over $N(S)\times N(T)$.

Theorem 14. Let (F,A) be a soft neutrosophic strong N-LA-semigroup over $S(N)$ and $\{(H_j,B_j): j \in J\}$ be a non-empty family of soft neutrosophic strong N-ideals of (F,A). Then

1. $\cap_{j\in J}^{R}(H_j,B_j)$ is a soft neutrosophic strong N-ideal of (F,A).
2. $\wedge_{j\in J}(H_j,B_j)$ is a soft neutrosophic strong N-ideal of (F,A).
3. $\cup_{j\in J}^{\mathcal{E}}(H_j,B_j)$ is a soft neutrosophic strong N-ideal of (F,A).
4. $\vee_{j\in J}(H_j,B_j)$ is a soft neutrosophic strong N-ideal of (F,A).

Conclusion

This paper we extend soft neutrosophic bisemigroup, soft neutrosophic N -semigroup to soft neutrosophic bi-LA-semigroup, and soft neutrosophic N -LA-semigroup. Their related properties and results are explained with many illustrative examples. The notions related with strong part of neutrosophy also established .

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Introduction to Image Processing via Neutrosophic Techniques

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Abstract. This paper is an attempt of proposing the processing approach of neutrosophic technique in image processing. As neutrosophic sets is a suitable tool to cope with imperfectly defined images, the properties, basic operations distance measure, entropy measures, of the neutrosophic sets method are presented here. İn this paper we, introduce the distances between neutrosophic sets: the Hamming distance, the normalized Hamming distance, the Euclidean distance and normalized Euclidean distance. We will extend the concepts of distances to the case of neutrosophic hesitancy degree. Entropy plays an important role in image processing. In our further considertions on entropy for neutrosophic sets the concept of cardinality of a neutrosophic set will also be useful. Possible applications to image processing are touched upon.

Keywords: Neutrosophic sets; Hamming distance; Euclidean distance; Normalized Euclidean distance; Image processing.

1. Introduction

Since the world is full of indeterminacy, the neutrosophics found their place into contemporary research. Smarandache [9, 10] and Salama et al [4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 45]. Entropy plays an important role in image processing. İn this paper we, introduce the distances between neutrosophic sets: the Hamming distance. İn this paper we, introduce the distances between neutrosophic sets: the Hamming distance, The normalized Hamming distance, the Euclidean distance and normalized Euclidean distance. We will extend the concepts of distances to the case of neutrosophic hesitancy degree. In our further considertions on entropy for neutrosophic sets the concept of cardinality of a neutrosophic set will also be useful.

2. Terminologies

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their classical and fuzzy counterparts [1, 2, 3, 11, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46] such as a neutrosophic set theory. We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [9, 10] and Salama et al. [4, 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 45]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]0^-,1^+[$ is nonstandard unit interval. Salama et al. introduced the following:

Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $A = \langle \mu_A(x), \sigma_A(x), \nu_A(x) \rangle$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ which represent the degree of member ship function (namely $\mu_A(x)$), the degree of indeterminacy (namely $\sigma_A(x)$), and the degree of non-member ship (namely $\nu_A(x)$) respectively of each element $x \in X$ to the set A where

$0^- \le \mu_A(x), \sigma_A(x), \nu_A(x) \le 1^+$ and

$0^- \le \mu_A(x) + \sigma_A(x) + \nu_A(x) \le 3^+$ .Smarandache introduced the following: Let T, I,F be real standard or nonstandard subsets of $\left]0^-,1^+\right[$, with

Sup_T=t_sup, inf_T=t_inf
Sup_I=i_sup, inf_I=i_inf
Sup_F=f_sup, inf_F=f_inf
n-sup=t_sup+i_sup+f_sup
n-inf=t_inf+i_inf+f_inf,
T, I, F are called neutrosophic components

3. Distances Betoween Neutrosophic Sets

We will now extend the concepts of distances presented in [11] to the case of neutrosophic sets.

Definition 3.1

Let $A = \{(\mu_A(x), \nu_A(x), \gamma_A(x)), x \in X\}$ and $B = \{(\mu_B(x), \nu_B(x), \gamma_B(x)), x \in X\}$ in $X = \{x_1, x_2, x_3, ..., x_n\}$ then

i) The Hamming distance is equal to

$$d_{Ns}(A,B) = \sum_{i=1}^{n} \left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\gamma_A(x_i) - \gamma_B(x_i)| \right)$$

.

ii) The Euclidean distance is equal to

$$e_{Ns}(A,B) = \sqrt{\sum_{i=1}^{n} \left((\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\gamma_A(x_i) - \gamma_B(x_i))^2 \right)}$$

iii) The normalized Hamming distance is equal to

$$NH_{Ns}(A,B) = \frac{1}{2n} \sum_{i=1}^{n} \left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\gamma_A(x_i) - \gamma_B(x_i)| \right)$$

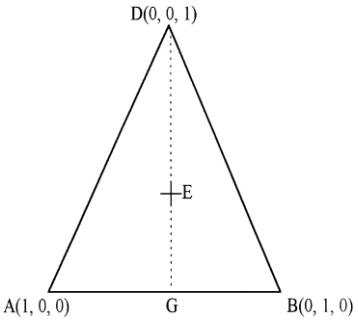
iv) The normalized Euclidean distance is equal to

$$NE_{Ns}(A,B) = \sqrt{\frac{1}{2n} \sum_{i=1}^{n} \left((\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\gamma_A(x_i) - \gamma_B(x_i))^2 \right)}$$

Example 3.1

Let us consider for simplicity degenrated neutrosophic sets A, B, D, G, F in $X = \{a\}$. A full description of each neutrosophic set i.e. $A = \{(\mu_A(x), \nu_A(x), \gamma_A(x)), a \in X\}$, may be exemplified by $A = \{\langle 1,0,0\rangle, a \in X\}$, $B = \{\langle 0,1,0\rangle, a \in X\}$, $D = \{\langle 0,0,1\rangle, a \in X\}$, $G = \{\langle 0.5,0.5,0\rangle, a \in X\}$, $E = \{\langle 0.25,0.25,0.0.5\rangle, a \in X\}$, .

Let us calculate four distances between the above neutrosophic sets using i), ii), iii) and iv) formulas ,



(Fig.1) A geometrical interpretation of the neutrosophic considered in Example 5.1 .

We obtain $e_{Ns}(A,D) = \frac{1}{2}$, $e_{Ns}(B,D) = \frac{1}{2}$, $e_{Ns}(A,B) = \frac{1}{2}$, $e_{Ns}(A,G) = \frac{1}{2}$, $e_{Ns}(B,G) = \frac{1}{2}$, $e_{Ns}(E,G) = \frac{1}{4}$, $e_{Ns}(D,G) = \frac{1}{4}$, $NE_{Ns}(A,B) = 1$, $NE_{Ns}(A,D) = 1$, $NE_{Ns}(B,D) = 1$, $NE_{Ns}(A,G) = \frac{1}{2}$, $NE_{Ns}(B,G) = \frac{1}{2}$, $NE_{Ns}(B,G) = \frac{1}{2}$, $NE_{Ns}(E,G) = \frac{\sqrt{3}}{4}$, and $NE_{Ns}(D,G) = \frac{\sqrt{3}}{2}$,

From the above results the triangle ABD (Fig.1) has edges equal to $\sqrt{2}$ and $e_{Ns}(A,D) = e_{Ns}(B,D) = e_{Ns}(A,B) = \frac{1}{2}$ and $NE_{Ns}(A,B) = NE_{Ns}(A,D) = NE_{Ns}(B,D) = 2NE_{Ns}(A,G) = 2NE_{Ns}(B,G) = 1$, and $NE_{Ns}(E,G)$ is equal to half of the height of triangle with all edges equal to $\sqrt{2}$ multiplied by, $\frac{1}{\sqrt{2}}$ i.e. $\frac{\sqrt{3}}{4}$.

Example 3.2

Let us consider the following neutrosophic sets A and B in $X = \{a, b, c, d, e\}$. ,

$A = \{\langle 0.5,0.3,0.2\rangle, \langle 0.2,0.6,0.2\rangle, \langle 0.3,0.2,0.5\rangle, \langle 0.2,0.2,0.6\rangle, \langle 1,0,0\rangle\}$

$B = \{\langle 0.2,0.6,0.2\rangle, \langle 0.3,0.2,0.5\rangle, \langle 0.5,0.2,0.3\rangle, \langle 0.9,0,0.1\rangle, \langle 0,0,0\rangle\}$

.Then $d_{Ns}(A,B) = 3$, $NH_{Ns}(A,B) = 0.43$, $e_{Ns}(A,B) = 1.49$ and $NE_{Ns}(A,B) = 0.55$.

Remark 3.1

Clearly these distances satisfy the conditions of metric space.

Remark 3.2

It is easy to notice that for formulas i), ii), iii) and iv) the following is valid:

a) $0 \le d_{Ns}(A,B) \le n$
b) $0 \le NH_{Ns}(A,B) \le 1$
c) $0 \le e_{Ns}(A,B) \le \sqrt{n}$
d) $0 \le NE_{Ns}(A,B) \le 1$.

This representation of a neutrosophic set (Fig. 2) will be a point of departure for neutrosophic crisp distances, and entropy of neutrosophic sets.

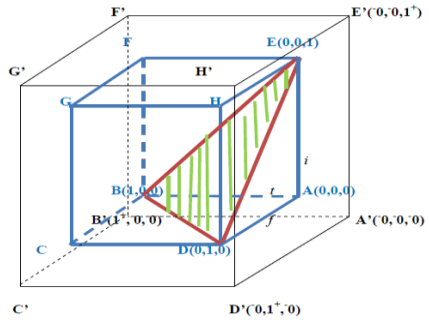


Fig. 2. A three-dimension representation of a neutrosophic set[9, 10].

4. Hesitancy Degree and Cardinality for Neutrosophic Sets

We will now extend the concepts of distances to the case of neutrosophic hesitancy degree. By taking into account the four parameters characterization of neutrosophic sets i.e. $A=\{< \mu_A(x), \nu_A(x), \gamma_A(x), \pi_A(x) >, x \in X\}$

Definition4.1

Let $A=\{(\mu_A(x), \nu_A(x), \gamma_A(x)), x \in X\}$ and $B=\{(\mu_B(x), \nu_B(x), \gamma_B(x)), x \in X\}$ on $X=\{x_1, x_2, x_3, ..., x_n\}$

For a neutrosophic set $A=\{(\mu_A(x), \nu_A(x), \gamma_A(x)), x \in X\}$ in X, we call $\pi_A(x)=3-\mu_A(x)-\nu_A(x)-\gamma_A(x)$, the neutrosophic index of x in A. İt is a hesitancy degree of x to A it is obvtous that $0 \le \pi_A(x) \le 3$.

Definition 4.2

Let $A=\{(\mu_A(x), \nu_A(x), \gamma_A(x)), x \in X\}$ and $B=\{(\mu_B(x), \nu_B(x), \gamma_B(x)), x \in X\}$ in $X=\{x_1, x_2, x_3, ..., x_n\}$ then

i) The Hamming distance is equal to

$$d_{Ns}(A,B)=\sum_{i=1}^{n}\left(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+|\gamma_A(x_i)-\gamma_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|\right)$$

. Taking into account that $\pi_A(x_i)=3-\mu_A(x_i)-\nu_A(x_i)-\gamma_A(x_i)$ and $\pi_B(x_i)=3-\mu_B(x_i)-\nu_B(x_i)-\gamma_B(x_i)$

we have

$$|\pi_A(x_i)-\pi_B(x_i)|=|3-\mu_A(x_i)-\nu_A(x_i)-\gamma_A(x_i)-3+\mu_A(x_i)+\nu_B(x_i)+\gamma_B(x_i)|$$
$$\le |\mu_B(x_i)-\mu_A(x_i)|+|\nu_B(x_i)-\nu_A(x_i)|+|\gamma_B(x_i)-\gamma_A(x_i)|$$

.

ii) The Euclidean distance is equal to

$$e_{Ns}(A,B)=\sqrt{\sum_{i=1}^{n}\left((\mu_A(x_i)-\mu_B(x_i))^2+(\nu_A(x_i)-\nu_B(x_i))^2+(\gamma_A(x_i)-\gamma_B(x_i))^2+(\pi_A(x_i)-\pi_B(x_i))^2\right)}$$

we have

$$(\pi_A(x_i)-\pi_B(x_i))^2 =$$
$$(-\mu_A(x_i)-\nu_A(x_i)-\gamma_A(x_i)+\mu_B(x_i)+\nu_B(x_i)+\gamma_B(x_i))^2$$
$$=(\mu_B(x_i)-\mu_A(x_i))^2+(\nu_A(x_i)-\nu_B(x_i))^2+$$
$$(\gamma_A(x_i)-\gamma_B(x_i))^2$$
$$+2(\mu_B(x_i)-\mu_A(x_i)(\nu_A(x_i)-\nu_B(x_i))$$
$$(\gamma_B(x_i)-\gamma_A(x_i))$$

iii) The normalized Hamming distance is equal to

$$NH_{Ns}(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\left(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+|\gamma_A(x_i)-\gamma_B(x_i)|+|\pi_A(x_i)-\pi_B(x_1)|\right)$$

iv) The normalized Euclidean distance is equal to

$$NE_{Ns}(A,B)=\sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left((\mu_A(x_i)-\mu_B(x_i))^2+(\nu_A(x_i)-\nu_B(x_i))^2+(\gamma_A(x_i)-\gamma_B(x_i))^2+(\pi_A(x_i)-\pi_B(x_i))^2\right)}$$

5.2 Remark

It is easy to notice that for formulas i), ii), iii) and the following is valid:

a) $0 \le d_{Ns}(A,B) \le 2n$

b) $0 \le NH_{Ns}(A,B) \le 2$

c) $0 \le e_{Ns}(A,B) \le \sqrt{2n}$

d) $0 \le NE_{Ns}(A,B) \le \sqrt{2}$.

5. from Images to Neutrosophic Sets, and Entropy

Given the definitions of the previous section several possible contributions are discussed. Neutrosophic sets may be used to solve some of the problems of data causes problems in the classification of pixels. Hesitancy in images originates from various factors, which in their majority are due to the inherent weaknesses of the acquisition and the imaging mechanisms. Limitations of the acquisition chain, such as the quantization noise, the suppression of the dynamic range, or the nonlinear behavior of the mapping system, affect our certainty on deciding whether a pixel is "*gray*" or "*edgy*" and therefore introduce a degree of hesitancy associated with the corresponding pixel. Therefore, hesitancy should encapsulate the aforementioned sources of indeterminacy that characterize digital images. Defining the membership component of the A–NS that describes the brightness of pixels in an image, is a more straightforward task that can be carried out in a similar manner as in traditional fuzzy image processing systems. In the presented heuristic framework, we consider the membership value of a gray level *g* to be its normalized

intensity level; that is $\mu_A(g) = \frac{g}{L-1_x}$ where $g \in \{0,..., L-1\}$.It should be mentioned that any other method for calculating $\mu_A(g)$ can also be applied.

In the image is A being (x, y) the coordinates of each pixel and the $g(x, y)$ be the gray level of the pixel (x, y) implies $0 \le g(x, y) \le L-1$. Each image pixel is associated with four numerical values:

- A value representing the membership $\mu_A(x)$, obtained by means of membership function associated with the set that represents the expert's knowledge of the image.
- A value representing the indeterminacy $\nu_A(x)$, obtained by means of the indeterminacy function associated with the set that represents the ignorance of the expert's decision.
- A value representing the non-membership $\gamma_A(x)$, obtained by means of the non -membership function associated with the set that represents the ignorance of the expert's decision.
- A value representing the hesitation measure $\pi_A(x)$, obtained by means of the $\pi_A(x) = 3 - \mu_A(x) - \nu_A(x) - \gamma_A(x)$.

Let an image A of size $M \times N$ pixels having L gray levels ranging between 0 and *L-1*. The image in the neutrosophic domain is considered as an array of neutrosophic singletons. Here, each element denoted the degree of the membership, indeterminacy and non-membership according to a pixel with respect to an image considered. An image A in neutrosophic set is $A = \{< \mu_A(g_{ij}), \nu_A(g_{ij}), \gamma_A(g_{ij}) >, g_{ij} \in \{0,..., L-1\}\}$ where $\mu_A(g_{ij}), \nu_A(g_{ij}), \gamma_A(g_{ij})$ denote the degrees of membership indeterminacy and non-membership of the $(i, j)-th$ pixel to the set A associated with an image property $\mu_A(g) = \frac{g - g_{\min}}{g - g_{\max}}$ where $g_{\min}$ and $g_{\max}$ are the minimum and the maximum gray levels of the image. Entropy plays an important role in image processing. In our further considertions on entropy for neutrosophic sets the concept of cardinality of a neutrosophic set will also be useful

Definition 5.1

Let $A = \langle (\mu_A(x), \nu_A(x), \gamma_A(x)), x \in X \rangle$ a neutrosophic set in X, first, we define two cardinalities of a neutrosophic set

- The least (sure) cadinality of A is equal to so is called segma-count, and is called here the $\min \sum cont(A) = \sum_{i=1} \mu_A(x_i) + \sum_{i=1} \nu_A(x_i)$
- The bigesst cadinality of A , which is possible due to $\pi_A(x)$ is equal to $\max \sum cont(A) = \sum_{i=1} (\mu_A(x_i) + \pi_A(x_i)) + \sum_{i=1} \nu_A(x_i) + \pi_A(x_i))$

and , clearly for A^c we have $\min \sum cont(A^c) = \sum_{i=1} \gamma_A(x_i) + \sum_{i=1} \nu_A(x_i)$,

$\max \sum cont(A^c) = \sum_{i=1} (\gamma_A(x_i) + \pi_A(x_i)) + \sum_{i=1} \nu_A(x_i) + \pi_A(x_i))$

. Then the cadinality of neutrosophic set is defined as the interval

$Card(A) = [\min \sum Cont(A), \max \sum Cont(A)]$

Definition 5.2

An entropy on $NS(X)$ is a real-valued functional $E : NS(X) \to [0,1]$, satisfying the following axiomatic requirements:

E_1: $E(A) = 0$ iff A is a neutrosophic crisp set; that is $\mu_A(x_i) = 0$ or $\mu_A(x_i) = 1$ for all $x_i \in X$.

E_2 : $E(A) = 1$ iff $\mu_A(x_i) = \nu_A(x_i) = \gamma_A(x_i)$ for all $x_i \in X$. that is $A = A^c$.

E_3: $E(A) \le E(B)$ if A refine B; i.e. $A \le B$.

E_4: $E(A) = E(A^c)$

Where a neutrosophic entropy measure be define as

$E(A) = \frac{1}{n} \sum_{i=1}^{n} \frac{\max Count(A_i \cap A_i^c)}{\max Count(A_i \cup A_i^c)}$ where

$n = Cardinal(X)$ and A_i denotes the single-element A–NS corresponding to the i[th] element of the universe X and is described as

$A_i = \{(\mu_A(x_i), \nu_A(x_i), \gamma_A(x_i)), x_i \in X\}$.

In other words, A_i is the i[th] "component" of A.

Moreover, $\max Count(A)$ denotes the biggest cardinality of A and is given by :

$\max \sum cont(A) = \sum_{i=1} (\mu_A(x_i) + \pi_A(x_i)) + \sum_{i=1} \nu_A(x_i) + \pi_A(x_i))$

Conclusion

Some of the properties of the neutrosophic sets, Distance measures, Hesitancy Degree, Cardinality and Entropy measures are briefed in this paper. These measures can be used effectively in image processing and pattern recognition. The future work will cover the application of these measures.

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Neutrosophic Soft Multi-Set Theory and Its Decision Making

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Abstract. In this study, we introduce the concept of neutrosophic soft multi-set theory and study their properties and operations. Then, we give a decision making methods for neutrosophic soft multi-set theory. Finally, an application of this method in decision making problems is presented.

Keywords: Soft set, neutrosophic set, neutrosophic refined set, neutrosophic soft multi-set, decision making.

1. Introduction

In 1999, a Russian researcher Molodtsov [23] initiated the concept of soft set theory as a general mathematical tool for dealing with uncertainty and vagueness. The theory is in fact a set-valued map which is used to describe the universe of discourse based on some parameters which is free from the parameterization inadequacy syndrome of fuzzy set theory [31], rough set theory [25], and so on. After Molodtsov's work several researchers were studied on soft set theory with applications (i.e [13,14,21]). Then, Alkhazaleh et al [3] presented the definition of soft multiset as a generalization of soft set and its basic operation such as complement, union, and intersection. Also, [6,7,22,24] are studied on soft multiset. Later on, in [2] Alkazaleh and Salleh introduced fuzzy soft set multisets, a more general concept, which is a combination of fuzzy set and soft multisets and studied its properties and gave an application of this concept in decision making problem. Then, Alhazaymeh and Hassan [1] introduce the concept of vague soft multisets which is an extension of soft sets and presented application of this concept in decision making problem. These concepts cannot deal with indeterminant and inconsistent information.

In 1995, Smarandache [26,30] founded a theory is called neutrosophic theory and neutrosophic sets has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in real world. The theory is a powerful tool which generalizes the concept of the classical set, fuzzy set [31], interval-valued fuzzy set [29], intuitionistic

fuzzy set [4], interval-valued intuitionistic fuzzy set [5], and so on.

Recently, Maji [20] proposed a hybrid structure is called neutrosophic soft set which is a combination of neutrosophic set [26] and soft sets [23] and defined several operations on neutrosophic soft sets and made a theoretical study on the theory of neutrosophic soft sets. After the introduction of neutrosophic soft set, many scholars have done a lot of good researches in this filed [8,9,11,18,19,27,28]. In recently, Deli [16] defined the notion of interval-valued neutrosophic soft set and interval-valued neutrosophic soft set operations to make more functional. After the introduction of interval-valued neutrosophic soft set Broumi et al. [10] examined relations of interval-valued neutrosophic soft set. Many interesting applications of neutrosophic set theory have been combined with soft sets in [12,17]. But until now, there have been no study on neutrosophic soft multisets. In this paper our main objective is to study the concept of neutrosophic soft multisets which is a combination of neutrosophic multi(refined) [15] set and soft multisets [3]. The paper is structured as follows. In Section 2, we first recall the necessary background material on neutrosophic sets and soft set. The concept of neutrosophic soft multisets and some of their properties are presented in Section 3. In Section 4, we present algorithm for neutrosophic soft multisets. In section 5 an application of neutrosophic soft multisets in decision making is presented. Finally we conclude the paper.

2. Preliminaries

Throughout this paper, let U be a universal set and E be the set of all possible parameters under consideration with respect to U, usually, parameters are attributes, characteristics, or properties of objects in U.

We now recall some basic notions of, neutrosophic set, soft set and neutrosophic soft sets. For more details, the reader could refer to [15,20,23,26,30].

Definition 2.1.[26] Let U be a universe of discourse then the neutrosophic set A is an object having the form

$$A = \{< x: \mu_{A(x)}, \nu_{A(x)}, \omega_{A(x)}>, x \in U\}$$

where the functions μ, ν, ω : $U \rightarrow]^{-}0,1^{+}[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in X$ to the set A with the condition.

$$^{-}0 \leq \mu_{A(x)} + \nu_{A(x)} + \omega_{A(x)} \leq 3^{+}.$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]-0,1^{+}[$. So instead of $]^{-}0,1^{+}[$ we need to take the interval [0,1] for technical applications cause $]^{-}0,1^{+}[$ will be difficult to apply in the real world applications such as in scientific and engineering problems.

For two NS,

$$_{NS} = \{<x, \mu_A(x), \nu_A(x), \omega_A(x)> \mid x \in X\}$$

and

$$B_{NS} = \{<x, \mu_B(x), \nu_B(x), \omega_B(x)> \mid x \in X\}$$

Set- theoretic operations;

1. The subset; $_{NS} \subseteq B_{NS}$ if and only if

$\mu_A(x) \leq \mu_B(x)$, $\nu_A(x) \geq \nu_B(x)$ and $\omega_A(x) \geq \omega_B(x)$.

2. $_{NS} = B_{NS}$ if and only if,

$\mu_A(x) = \mu_B(x)$,$\nu_A(x) = \nu_B(x)$ and $\omega_A(x) = \omega_B(x)$

for any $x \in X$.

3. The complement of $_{NS}$ is denoted by $^{o}_{NS}$ and is defined by

$$^{o}_{NS} = \{<x, \omega_A(x), 1 - \nu_A(x), \mu_A(x) \mid x \in X\}$$

4. The intersection

$$A \cap B = \{<x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\}, \max\{\omega_A(x), \omega_B(x)\}> : x \in X\}$$

5. The union

$$A \cup B = \{<x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\}, \min\{\omega_A(x), \omega_B(x)\}> : x \in X\}$$

Definition 2.2 [23] Let U be an initial universe set and E be a set of parameters. Let P (U) denotes the power set of U. Consider a nonempty set A, $A \subset E$. A pair (K, A) is called a soft set over U, where K is a mapping given by $K: A \rightarrow P(U)$.

For an illustration, let us consider the following example.

Example 2.3. Suppose that U is the set of houses under consideration, say $U = \{h_1, h_2, \ldots, h_{10}\}$. Let E be the set of some attributes of such houses, say $E = \{e_1, e_2, \ldots, e_4\}$, where $e_1, e_2, \ldots, e_4$ stand for the attributes "beautiful", "costly", "in the green surroundings'", "moderate", respectively. In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. For example, the soft set (K, A) that describes the "attractiveness of the houses" in the opinion of a buyer, says Mrs X, may be defined like this:

$A = \{e_1, e_2, e_3, e_4\}$;

$K(e_1) = \{h_1, h_3, h_7\}$, $K(e_2) = \{h_2\}$, $K(e_3) = \{h_{10}\}$, $K(e_4) = U$

Definition 2.4[20] Let **U** be an initial universe set and $\mathbf{A} \subset \mathbf{E}$ be a set of parameters. Let NS (U) denotes the set of all neutrosophic subsets of **U**. The collection $(\mathbf{F}, \mathbf{A})$ is termed to be the neutrosophic soft set over **U**, where **F** is a mapping given by $\mathbf{F}: \mathbf{A} \rightarrow \mathbf{NS}(\mathbf{U})$.

Example 2.5 [20] Let U be the set of houses under consideration and E is the set of parameters. Each parameter is a neutrosophic word or sentence involving neutrosophic words. Consider $E = \{$beautiful, wooden, costly, very costly, moderate, green surroundings, in good repair, in bad repair, cheap, expensive$\}$. In this case, to define a neutrosophic soft set means to point out beautiful houses, wooden houses, houses in the green surroundings and so on. Suppose that, there are five houses in the universe U given by $U = \{h_1, h_2, \ldots, h_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$,where e_1 stands for the parameter `beautiful', $e_2$ stands for the parameter `wooden', e_3 stands for the parameter `costly' and the parameter $e_4$stands for `moderate'. Then the neutrosophic soft set (F, A) is defined as follows:

$$(F,A)=\left\{\begin{array}{l}\left(e_1,\left\{\frac{h_1}{(0.5,0.6,0.3)},\frac{h_2}{(0.4,0.7,0.6)},\frac{h_3}{(0.6,0.2,0.3)},\frac{h_4}{(0.7,0.3,0.2)},\frac{h_5}{(0.8,0.2,0.3)}\right\}\right),\\ \left(e_2,\left\{\frac{h_1}{(0.6,0.3,0.5)},\frac{h_2}{(0.7,0.4,0.3)},\frac{h_3}{(0.8,0.1,0.2)},\frac{h_4}{(0.7,0.1,0.3)},\frac{h_5}{(0.8,0.3,0.6)}\right\}\right),\\ \left(e_3,\left\{\frac{h_1}{(0.7,0.4,0.3)},\frac{h_2}{(0.6,0.7,0.2)},\frac{h_3}{(0.7,0.2,0.5)},\frac{h_4}{(0.5,0.2,0.6)},\frac{h_5}{(0.7,0.3,0.4)}\right\}\right),\\ \left(e_4,\left\{\frac{h_1}{(0.8,0.6,0.4)},\frac{h_2}{(0.7,0.9,0.6)},\frac{h_3}{(0.7,0.6,0.4)},\frac{h_4}{(0.7,0.8,0.6)},\frac{h_5}{(0.9,0.5,0.7)}\right\}\right)\end{array}\right\}$$

3-Neutrosophic Soft Multi-Set Theory

In this section, we introduce the definition of a neutrosophic soft multi-set(Nsm-set) and its basic operations such as complement, union and intersection with examples. Some of it is quoted from [1,2,3, 6,7,22,24].

Obviously, some definitions and examples are an extension of soft multi-set [3] and fuzzy soft multi-sets [2].

Definition 3.1. Let $\{U_i : i \in I\}$be a collection of universes such that $\bigcap_{i\in I} U_i = \Phi$, $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters, $U=\prod_{i\in I} NSM(U_i)$ where $NSM(U_i)$ denotes the set of all NSM-subsets of U_i and $E=\prod_{i\in I} E_{U_i}$ and $\subseteq E$. Then, N_A is a neutrosophic soft multi-set (Nsm-set) over U, where N_A is a mapping given by N_A: A→ U.

Thus, a Nsm-set N_A over U can be represented by the set of ordered pairs.

$N_A=\{(x_1, N_A(x_1)) : x_1 \in \subseteq E\}$.

To illustrate this let us consider the following example:

Example 3.2 Suppose that Mr. X has a budget to buy a house, a car and rent a venue to hold a wedding celebration. Let us consider a Nsm-set $N_A$which describes "houses," "cars," and "hotels" that Mr.X is considering for accommodation purchase, transportation-purchase, and a venue to hold a wedding celebration, respectively.

Assume that $U_1=\{u_1,u_2,u_3,u_4\}$, $U_2=\{c_1,c_2,c_3,c_4\}$ and $U_3=\{h_1,h_2,h_3\}$ are three universal set and

$E_1=\{x_1{}^{U_1}=$ expensiv,$x_2{}^{U_1}=$ cheap, $x_3{}^{U_1}=$ wooden$\}$,

$E_2=\{x_1{}^{U_2}=$ expensive,$x_2{}^{U_2}=$ in green surroundings, $x_3{}^{U_2}=$ sporty$\}$ and

$E_3=\{x_1{}^{U_3}=$expensive,$x_2{}^{U_3}=$ majestic, $x_3{}^{U_3}=$ in Kuala Lumpur$\}$

Three parameter sets that is a collection of sets of decision parameters related to the above universes.

Let $U=\prod_1^3 NSM(U_i)$ and $E=\prod_1^3 E_{U_i}$ and $\subseteq E$ such that

$$A=\{x_1=\{x_1{}^{U_1},x_1{}^{U_2},x_1{}^{U_3}\}, x_2=\{x_2{}^{U_1},x_2{}^{U_2},x_2{}^{U_3}\}\}$$

and

$$N_A(x_1)=\left\{\left\{\frac{u_1}{(.5,.3,.4)},\frac{u_2}{(.2,.4,.4)},\frac{u_3}{(.3,.3,.5)},\frac{u_4}{(.7,.8,.4)}\right\},\left\{\frac{c_1}{(.7,.1,.5)},\frac{c_2}{(.2,.5,.7)},\frac{c_3}{(.7,.8,.0)},\frac{c_4}{(.0,.0,.0)}\right\},\left\{\frac{h_1}{(.0,.0,.0)},\frac{h_2}{(1,1,.0)},\frac{h_3}{(.9,.2,.5)}\right\}\right\}$$

$$N_A(x_2)=\left\{\left\{\frac{u_1}{(.1,.5,.3)},\frac{u_2}{(.1,.8,.9)},\frac{u_3}{(.0,.0,1)},\frac{u_4}{(.2,.8,.5)}\right\},\left\{\frac{c_1}{(.5,.5,.5)},\frac{c_2}{(.5,.3,.7)},\frac{c_3}{(.5,.4,.3)},\frac{c_4}{(.1,1,1)}\right\},\left\{\frac{h_1}{(1,.2,.5)},\frac{h_2}{(1,1,1)},\frac{h_3}{(.1,.8,.6)}\right\}\right\}$$

Then a Nsm-set N_A is written by

$$N_A=\left\{\left(x_1,\left(\left\{\frac{u_1}{(.5,.3,.4)},\frac{u_2}{(.2,.4,.4)},\frac{u_3}{(.3,.3,0.5)},\frac{u_4}{(.7,.8,.4)}\right\},\left\{\frac{c_1}{(.7,.1,.5)},\frac{c_2}{(.2,.5,.7)},\frac{c_3}{(.7,.8,.0)},\frac{c_4}{(.0,.0,.0)}\right\},\left\{\frac{h_1}{(.0,.0,.0)},\frac{h_2}{(1,1,.0)},\frac{h_3}{(.9,.2,.5)}\right\}\right)\right),\left(x_2,\left(\left\{\frac{u_1}{(.1,.5,.3)},\frac{u_2}{(.1,.8,.9)},\frac{u_3}{(.0,.0,1)},\frac{u_4}{(.2,.8,.5)}\right\},\left\{\frac{c_1}{(.5,.5,.5)},\frac{c_2}{(.5,.3,.7)},\frac{c_3}{(.5,.4,.3)},\frac{c_4}{(.1,1,1)}\right\},\left\{\frac{h_1}{(1,.2,.5)},\frac{h_2}{(1,1,1)},\frac{h_3}{(.1,.8,.6)}\right\}\right)\right)\right\}$$

Definition 3.3. Let N_A be a Nsm-set. Then, a pair $(x_i^{U_j}, N_A(x_i^{U_j}))$ is called an U_i-Nsm-set part,
$x_i^{U_j} \in x_k$ and $N_A(x_i^{U_j}) \subseteq N_A(x_i)$ such that $x_k \in \{x_1, x_2, \dots, x_n\}$, $i \in \{1,2,\dots,m\}$ and $j \in \{1,2,\dots,r\}$.

Example 3.4. Consider Example 3.2. Then,

$$(x_i^{U_1}, N_A(x_i^{U_1})) = \left\{\left(x_1^{U_1}, \left\{\frac{u_1}{(0.5,0.3,0.4)}, \frac{u_2}{(0.2,0.4,0.4)}, \frac{u_3}{(0.3,0.3,0.5)}, \frac{u_4}{(0.7,0.8,0.4)}\right\}\right), \left(x_2^{U_1}, \left\{\frac{u_1}{(0.1,0.5,0.3)}, \frac{u_2}{(0.1,0.8,0.9)}, \frac{u_3}{(0.0,0.0,1.0)}, \frac{u_4}{(0.2,0.8,0.5)}\right\}\right)\right\}$$

is a U_1-Nsm-set part of N_A.

Definition 3.5. Let N_A and N_B be a Nsm-sets. Then, N_A is NSMS-subset of N_B, denoted by $N_A \sqsubseteq N_B$ if and only if $N_A(x_I^{U_j})$ is a neut − rosophic subset of $N_B(x_i^{U_j})$ for all $x_I^{U_j} \in x_k$ *such that* $x_k \in \{x_1, x_2, \dots, x_n\}$, $i \in \{1,2,\dots,m\}$ and $j \in \{1,2,\dots,r\}$.

Example3.4. Let
A=$\{x_1 = \{x_1^{U_1}, x_1^{U_2}, x_1^{U_3}\}, x_2 = \{x_2^{U_1}, x_2^{U_2}, x_2^{U_3}\}\}$ and
B=$\{x_1 = \{x_1^{U_1}, x_1^{U_2}, x_1^{U_3}\}, x_2 = \{x_2^{U_1}, x_2^{U_2}, x_2^{U_3}\}$
$x_3 = \{x_3^{U_1}, x_3^{U_2}, x_3^{U_3}\}\}$

Clearly A⊆ B . Let N_A and N_B be two Nsm-set over the same U such that

$$N_A = \left\{\left(x_1, \left(\left\{\frac{u_1}{(0.5,0.3,0.4)}, \frac{u_2}{(0.2,0.4,0.4)}, \frac{u_3}{(0.3,0.3,0.5)}, \frac{u_4}{(0.7,0.8,0.4)}\right\}, \left\{\frac{c_1}{(0.7,0.1,0.5)}, \frac{c_2}{(0.2,05,0.7)}, \frac{c_3}{(0.7,0.8,0.0)}, \frac{c_4}{(0.0,0.0,0.0)}\right\} \left\{\frac{h_1}{(0.0,0.0,0.0)}, \frac{h_2}{(1.0,1.0,0.0)}, \frac{h_3}{(0.9,0.2,0.5)}\right\}\right)\right),$$
$$\left(x_2, \left(\left\{\frac{u_1}{(0.1,0.5,0.3)}, \frac{u_2}{(0.1,0.8,0.9)}, \frac{u_3}{(0.0,0.0,1.0)}, \frac{u_4}{(0.2,0.8,0.5)}\right\}, \left\{\frac{c_1}{(0.5,0.5,0.5)}, \frac{c_2}{(0.5,0.3,0.7)}, \frac{c_3}{(0.5,0.4,0.3)}, \frac{c_4}{(0.1,1.0,1.0)}\right\}, \left\{\frac{h_1}{(1.0,0.2,0.5)}, \frac{h_2}{(1.0,1.0,1.0)}, \frac{h_3}{(0.1,0.8,0.6)}\right\}\right)\right)\right\}$$

$$N_B = \left\{\left(x_1, \left(\left\{\frac{u_1}{(0.6,0.1,0.2)}, \frac{u_2}{(0.3,0.3,0.3)}, \frac{u_3}{(0.7,0.2,0.4)}, \frac{u_4}{(0.8,0.6,0.3)}\right\}, \left\{\frac{c_1}{(0.9,0.1,0.4)}, \frac{c_2}{(0.3,0.7,0.6)}, \frac{c_3}{(0.8,0.4,0.0)}, \frac{c_4}{(1.0,0.0,0.0)}\right\}, \left\{\frac{h_1}{(1.0,0.0,0.0)}, \frac{h_2}{(0.9,0.7,0.0)}, \frac{h_3}{(1.0,0.0,0.0)}\right\}\right)\right),$$
$$\left(x_2, \left(\left\{\frac{u_1}{(0.8,0.3,0.2)}, \frac{u_2}{(0.7,0.6,0.4)}, \frac{u_3}{(0.8,0.0,0.7)}, \frac{u_4}{(0.5,0.6,0.3)}\right\}, \left\{\frac{c_1}{(0.6,0.4,0.3)}, \frac{c_2}{(0.7,0.2,0.6)}, \frac{c_3}{(0.6,0.1,0.2)}, \frac{c_4}{(1.0,0.3,0.1)}\right\}, \left\{\frac{h_1}{(1.0,0.0,0.0)}, \frac{h_2}{(1.0,0.0,0.1)}, \frac{h_3}{(0.8,0.3,0.4)}\right\}\right)\right),$$
$$\left(x_3, \left(\left\{\frac{u_1}{(0.5,0.6,0.4)}, \frac{u_2}{(0.2,0.7,0.5)}, \frac{u_3}{(0.3,0.9,0.3)}, \frac{u_4}{(0.2,0.8,0.7)}\right\}, \left\{\frac{c_1}{(0.8,0.3,0.5)}, \frac{c_2}{(0.8,0.3,0.1)}, \frac{c_3}{(0.3,0.5,0.6)}, \frac{c_4}{(0.9,0.3,0.2)}\right\}, \left\{\frac{h_1}{(0.3,0.8,0.6)}, \frac{h_2}{(0.0,1.0,0.2)}, \frac{h_3}{(0.3,0.6,0.5)}\right\}\right)\right)\right\}$$

Then, we have $N_A \subseteq N_B$.

Definition 3.6. Let N_A and N_B are two Nsm-sets. Then, $N_A = N_B$, if and only if $N_A \subseteq N_B$ and $N_B \subseteq N_A$.

Definition 3.7. Let N_A be a Nsm-set. Then, the complement of N_A, denoted by N_A^c , is defined by
$N_A^c = \{(x, N_A^o(x)) : x \in \subseteq E\}$
where $N_A^o(x)$ is a NM complement.

Example3.4.

$$N_A^o(x) = \left\{\left(x_1, \left(\left\{\frac{u_1}{(0.4,0.7,0.5)}, \frac{u_2}{(0.4,0.6,0.2)}, \frac{u_3}{(0.5,0.7,0.3)}, \frac{u_4}{(0.4,0.2,0.7)}\right\}, \left\{\frac{c_1}{(0.5,0.9,0.7)}, \frac{c_2}{(0.7,0.5,0.2)}, \frac{c_3}{(0.0,0.2,0.7)}, \frac{c_4}{(0.0,1.0,0.0)}\right\}, \left\{\frac{h_1}{(0.0,1.0,0.0)}, \frac{h_2}{(0.0,9.0,1.0)}, \frac{h_3}{(0.5,0.8,0.9)}\right\}\right)\right),$$
$$\left(x_2, \left(\left\{\frac{u_1}{(0.3,0.5,0.1)}, \frac{u_2}{(0.9,0.2,0.1)}, \frac{u_3}{(1.0,1.0,0.0)}, \frac{u_4}{(0.5,0.2,0.2)}\right\}, \left\{\frac{c_1}{(0.5,0.5,0.5)}, \frac{c_2}{(0.7,0.7,0.5)}, \frac{c_3}{(0.3,0.6,0.5)}, \frac{c_4}{(1.0,0.0,0.1)}\right\}, \left\{\frac{h_1}{(0.5,0.8,1.0)}, \frac{h_2}{(1.0,0.0,1.0)}, \frac{h_3}{(0.6,0.2,0.1)}\right\}\right)\right)\right\}$$

Definition 3.8. A Nsm-set N_A over U is called a null Nsm-set, denoted by $N_{A\emptyset}$ if all of the Nsm-set parts of $N_A$equals ∅.

Example3.4. Consider Example 3.2 again, with a Nsm-set N_A which describes the ”at-

tractiveness of stone houses", "cars" and "hotels". Let

$A=\{x_1=\{x_1{}^{U_1},x_1{}^{U_2},x_1{}^{U_3}\},x_2=\{x_2{}^{U_1},x_2{}^{U_2},x_2{}^{U_3}\}\}$.

The Nsm-set N_A is the collection of approximations as below:

$$A_\emptyset = \left\{\left(x_1,\left(\left\{\frac{u_1}{(0.0,1.0,1.0)},\frac{u_2}{(0.0,1.0,1.0)},\frac{u_3}{(0.0,1.0,1.0)},\frac{u_4}{(0.0,1.0,1.0)}\right\},\left\{\frac{c_1}{(0.0,1.0,1.0)},\frac{c_2}{(0.0,1.0,1.0)},\frac{c_3}{(0.0,1.0,1.0)},\frac{c_4}{(0.0,1.0,1.0)}\right\},\left\{\frac{h_1}{(0.0,1.0,1.0)},\frac{h_2}{(0.0,1.0,1.0)},\frac{h_3}{(0.0,1.0,1.0)}\right\}\right)\right.,$$
$$\left.\left(x_2,\left(\left\{\frac{u_1}{(0.0,1.0,1.0)},\frac{u_2}{(0.0,1.0,1.0)},\frac{u_3}{(0.0,1.0,1.0)},\frac{u_4}{(0.0,1.0,1.0)}\right\},\left\{\frac{c_1}{(0.0,1.0,1.0)},\frac{c_2}{(0.0,1.0,1.0)},\frac{c_3}{(0.0,1.0,1.0)},\frac{c_4}{(0.0,1.0,1.0)}\right\},\left\{\frac{h_1}{(0.0,1.0,1.0)},\frac{h_2}{(0.0,1.0,1.0)},\frac{h_3}{(0.0,1.0,1.0)}\right\}\right)\right)\right\}$$

Then, $A_\emptyset$ is a null Nsm-set.

Definition 3.8. A Nsm-set A over U is called a semi-null Nsm-set, denoted by $N_{A_{\approx\emptyset}}$ if at least all the Nsm-set parts of $N_{A_{\approx\emptyset}}$ equals $\emptyset$.

Example3.4. Consider Example 3.2 again, with a Nsm-set N_A which describes the "attractiveness of stone houses", "cars" and "hotels". Let

$A=\{x_1=\{x_1{}^{U_1},x_1{}^{U_2},x_1{}^{U_3}\},x_2=\{x_2{}^{U_1},x_2{}^{U_2},x_2{}^{U_3}\}\}$.

The Nsm-set A is the collection of approximations as below:

$$A_{\approx\emptyset} = \left\{\left(x_1,\left(\left\{\frac{u_1}{(0.0,1.0,1.0)},\frac{u_2}{(0.0,1.0,1.0)},\frac{u_3}{(0.0,1.0,1.0)},\frac{u_4}{(0.0,1.0,1.0)}\right\},\left\{\frac{c_1}{(0.5,0.9,0.7)},\frac{c_2}{(0.7,0.5,0.2)},\frac{c_3}{(0.0,0.2,0.7)},\frac{c_4}{(0.0,1.0,0.0)}\right\},\left\{\frac{h_1}{(0.0,1.0,0.0)},\frac{h_2}{(0.0,9.0,1.0)},\frac{h_3}{(0.5,0.8,0.9)}\right\}\right)\right.,$$
$$\left.\left(x_2,\left(\left\{\frac{u_1}{(0.0,1.0,1.0)},\frac{u_2}{(0.0,1.0,1.0)},\frac{u_3}{(0.0,1.0,1.0)},\frac{u_4}{(0.0,1.0,1.0)}\right\},\left\{\frac{c_1}{(0.5,0.5,0.5)},\frac{c_2}{(0.7,0.7,0.5)},\frac{c_3}{(0.3,0.6,0.5)},\frac{c_4}{(1.0,0.0,0.1)}\right\},\left\{\frac{h_1}{(0.5,0.8,1.0)},\frac{h_2}{(1.0,0.0,1.0)},\frac{h_3}{(0.6,0.2,0.1)}\right\}\right)\right)\right\}$$

Then $A_{\approx\emptyset}$ is a semi null Nsm-set

Definition 3.8. A Nsm-set N_A over U is called a semi-absolute Nsm-set, denoted by $A_{\approx U_i}$ if $N_A(x_i{}^{U_j})=U_i$ for at least one $x_k\in\{x_1,x_2,\dots,x_n\}$, $i\in\{1,2,\dots,m\}$ and $j\in\{1,2,\dots,r\}$.

Example3.4. Consider Example 3.2 again, with a Nsm-set N_A which describes the "attractiveness of stone houses", "cars" and "hotels". Let

$A=\{x_1=\{x_1{}^{U_1},x_1{}^{U_2},x_1{}^{U_3}\},x_2=\{x_2{}^{U_1},x_2{}^{U_2},x_2{}^{U_3}\}\}$.

The Nsm-set N_A is the collection of approximations as below:

$$A_{\approx U_i} = \left\{\left(x_1,\left(\left\{\frac{u_1}{(1.0,0.0,0.0)},\frac{u_2}{(1.0,0.0,0.0)},\frac{u_3}{(1.0,0.0,0.0)},\frac{u_4}{(1.0,0.0,0.0)}\right\},\left\{\frac{c_1}{(0.5,0.9,0.7)},\frac{c_2}{(0.7,0.5,0.2)},\frac{c_3}{(0.0,0.2,0.7)},\frac{c_4}{(0.0,1.0,0.0)}\right\},\left\{\frac{h_1}{(0.0,1.0,0.0)},\frac{h_2}{(0.0,9.0,1.0)},\frac{h_3}{(0.5,0.8,0.9)}\right\}\right)\right.,$$
$$\left.\left(x_2,\left(\left\{\frac{u_1}{(1.0,0.0,0.0)},\frac{u_2}{(1.0,0.0,0.0)},\frac{u_3}{(1.0,0.0,0.0)},\frac{u_4}{(1.0,0.0,0.0)}\right\},\left\{\frac{c_1}{(0.5,0.5,0.5)},\frac{c_2}{(0.7,0.7,0.5)},\frac{c_3}{(0.3,0.6,0.5)},\frac{c_4}{(1.0,0.0,0.1)}\right\},\left\{\frac{h_1}{(0.5,0.8,1.0)},\frac{h_2}{(1.0,0.0,1.0)},\frac{h_3}{(0.6,0.2,0.1)}\right\}\right)\right)\right\}$$

Then, $A_{\approx U_i}$ is a semi-absolute Nsm-set.

Definition 3.8. A Nsm-set A over U is called an absolute Nsm-set, denoted by A_{U_i} if $A(x_i{}^{U_j})=U_i$ for all i.

Example 3.4. Consider Example 3.2 again, with a Nsm-set N_A which describes the "attractiveness of stone houses", "cars" and "hotels". Let

$A=\{x_1=\{x_1{}^{U_1},x_1{}^{U_2},x_1{}^{U_3}\},x_2=\{x_2{}^{U_1},x_2{}^{U_2},x_2{}^{U_3}\}\}$.

The Nsm-set N_A is the collection of approximations as below:

$$A_{U_i}=\{(x_1,(\{\frac{u_1}{(1,.0,.0)},\frac{u_2}{(1,.0,.0)},\frac{u_3}{(1,.0,.0)},\frac{u_4}{(1,.0,.0)}\},\{\frac{c_1}{(1,.0,.0)},\frac{c_2}{(1,.0,.0)},\frac{c_3}{(1,.0,.0)},\frac{c_4}{(1,.0,.0)}\},\{\frac{h_1}{(1,.0,.0)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(1,.0,.0)}\})),(x_2,(\{\frac{u_1}{(1,.0,.0)},\frac{u_2}{(1,.0,.0)},\frac{u_3}{(1,.0,.0)},\frac{u_4}{(1,.0,.0)}\},\{\frac{c_1}{(1,.0,.0)},\frac{c_2}{(1,.0,.0)},\frac{c_3}{(1,.0,.0)},\frac{c_4}{(1,.0,.0)}\},\{\frac{h_1}{(1,.0,.0)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(1,.0,.0)}\}))\}$$

Then, A_{U_i} is an absolute Nsm-set.

Proposition 3.15. Let A, N_B and N_C are three Nsm-sets. Then

i. $(N_A^c)^c=N_A$
ii. $(A_{\approx\emptyset})^c=N_{A\approx U_i}$
iii. $(A_{\emptyset})^c=N_{AU_i}$
iv. $(A_{\approx U_i})^c=N_{A\approx\emptyset}$
v. $(A_{U_i})^c=N_{A\emptyset}$

Proof: The proof is straightforward

Definition 3.8. Let N_A and N_B are two Nsm-sets. Then, union of A and N_B denoted by $N_A \sqcup N_B$, is defined by

$N_A \sqcup N_B=\{(x_i, N_A(x_i) \cup N_B(x_i)): x_i \in E\}$

where $\cup$ *is a NS union,* $i \in \{1,2,\dots,m\}$ and $j \in \{1,2,\dots,r\}$.

Example 3.10.

Let

$A=\{x_1=\{x_1{}^{U_1},x_1{}^{U_2},x_1{}^{U_3}\}, x_2=\{x_2{}^{U_1},x_2{}^{U_2},x_2{}^{U_3}\}\}$ and

$B=\{x_1=\{x_1{}^{U_1},x_1{}^{U_2},x_1{}^{U_3}\},x_2=\{x_2{}^{U_1},x_2{}^{U_2},x_2{}^{U_3}\}$
$x_3=\{x_3{}^{U_1},x_3{}^{U_2},x_3{}^{U_3}\}\}$

$$N_A=\{(x_1,(\{\frac{u_1}{(.5,.3,.4)},\frac{u_2}{(.2,.4,.4)},\frac{u_3}{(.3,.3,.5)},\frac{u_4}{(.7,.8,.4)}\},\{\frac{c_1}{(.7,.1,.5)},\frac{c_2}{(.2,.5,.7)},\frac{c_3}{(.7,.8,.0)},\frac{c_4}{(.0,.0,.0)}\},\{\frac{h_1}{(.0,.0,.0)},\frac{h_2}{(1,1,.0)},\frac{h_3}{(.9,.2,.5)}\})),(x_2,(\{\frac{u_1}{(.1,.5,.3)},\frac{u_2}{(.1,.8,.9)},\frac{u_3}{(.0,.0,1)},\frac{u_4}{(.2,.8,.5)}\},\{\frac{c_1}{(.5,.5,.5)},\frac{c_2}{(.5,.3,.7)},\frac{c_3}{(.5,.4,.3)},\frac{c_4}{(.1,1,1)}\},\{\frac{h_1}{(1,.2,.5)},\frac{h_2}{(1,1,1)},\frac{h_3}{(.1,.8,.6)}\}))\},$$

$$N_B=\{(x_1,(\{\frac{u_1}{(.3,.7,.2)},\frac{u_2}{(.4,.3,.8)},\frac{u_3}{(.6,.5,.4)},\frac{u_4}{(.6,.7,.4)}\},\{\frac{c_1}{(.5,.6,.8)},\frac{c_2}{(.5,.7,.8)},\frac{c_3}{(.3,.5,.6)},\frac{c_4}{(1,.0,.0)}\},\{\frac{h_1}{(1,.0,.1)},\frac{h_2}{(.5,.6,.3)},\frac{h_3}{(1,.0,.0)}\})),(x_2,(\{\frac{u_1}{(.7,.3,.5)},\frac{u_2}{(.6,.7,.8)},\frac{u_3}{(.6,.8,.6)},\frac{u_4}{(.6,.7,.3)}\},\{\frac{c_1}{(.4,.3,.2)},\frac{c_2}{(.5,.6,.7)},\frac{c_3}{(.9,.1,.3)},\frac{c_4}{(1,.2,.1)}\},\{\frac{h_1}{(1,.0,.0)},\frac{h_2}{(1,.0,.1)},\frac{h_3}{(.4,.2,.3)}\})),(x_3,(\{\frac{u_1}{(.6,.3,.6)},\frac{u_2}{(.3,.2,.6)},\frac{u_3}{(.6,.7,.5)},\frac{u_4}{(.3,.7,.6)}\},\{\frac{c_1}{(.7,.5,.3)},\frac{c_2}{(.6,.7,.2)},\frac{c_3}{(.5,.4,.5)},\frac{c_4}{(.3,.6,.5)}\},\{\frac{h_1}{(.3,.5,.6)},\frac{h_2}{(1,.0,.0)},\frac{h_3}{(.3,.2,.7)}\}))\}$$

$$N_A \sqcup N_B=\{(x_1,(\{\frac{u_1}{(0.5,0.3,0.4)},\frac{u_2}{(0.4,0.3,0.4)},\frac{u_3}{(0.6,0.3,0.4)},\frac{u_4}{(0.7,0.7,0.4)}\},\{\frac{c_1}{(0.7,0.1,0.5)},\frac{c_2}{(0.5,0.5,0.7)},\frac{c_3}{(0.7,0.3,0.0)},\frac{c_4}{(1.0,0.0,0.0)}\},\{\frac{h_1}{(1.0,0.0,0.0)},\frac{h_2}{(1.0,0.1,0.0)},\frac{h_3}{(0.9,0.2,0.5)}\})),(x_2,(\{\frac{u_1}{(0.7,0.3,0.5)},\frac{u_2}{(0.6,0.7,0.8)},\frac{u_3}{(0.6,0.0,0.6)},\frac{u_4}{(0.6,0.7,0.3)}\},\{\frac{c_1}{(0.5,0.3,0.2)},\frac{c_2}{(0.5,0.3,0.7)},\frac{c_3}{(0.9,0.1,0.3)},\frac{c_4}{(1.0,0.2,0.1)}\},\{\frac{h_1}{(1.0,0.0,0.0)},\frac{h_2}{(1.0,0.0,0.1)},\frac{h_3}{(0.4,0.2,0.3)}\})),(x_3,(\{\frac{u_1}{(0.6,0.3,0.6)},\frac{u_2}{(0.3,0.2,0.6)},\frac{u_3}{(0.6,0.7,0.5)},\frac{u_4}{(0.3,0.7,0.6)}\},\{\frac{c_1}{(0.7,0.5,0.3)},\frac{c_2}{(0.6,0.7,0.2)},\frac{c_3}{(0.5,0.4,0.5)},\frac{c_4}{(0.3,0.6,0.5)}\},\{\frac{h_1}{(0.3,0.5,0.6)},\frac{h_2}{(1.0,0.0,0.0)},\frac{h_3}{(0.3,0.2,0.7)}\}))\}$$

Proposition 3.15. Let A, N_B and N_C are three Nsm-sets. Then

i. $N_A \sqcup (N_B \sqcup N_C)=(N_A \sqcup N_B) \sqcup N_C$
ii. $N_A \sqcup N_A = N_A$
iii. $N_A \sqcup N_{A\emptyset} = N_A$
iv. $N_A \sqcup N_{B\emptyset} = N_A$

Proof: The proof is straightforward

Definition 3.8. Let N_A and N_B are two Nsm-sets. Then, intersection of N_A and N_B, denoted by $N_A \sqcap N_B$, is defined by

$$N_A \sqcap N_B = \{(x_i, N_A(x_i) \cap N_B(x_i)) : x_i \in E\}$$

where $\cap$ *is a NS intersect*ion,
$i \in \{1,2, \dots ,m\}$ and $j \in \{1,2, \dots ,r\}$.

Example 3.10.

$$N_A = \{(x_1, (\{\frac{u_1}{(.5,.3,.4)}, \frac{u_2}{(.2,.4,.4)}, \frac{u_3}{(.3,.3,.5)}, \frac{u_4}{(.7,.8,.4)}\}, \{\frac{c_1}{(.7,.1,.5)}, \frac{c_2}{(.2,.5,.7)}, \frac{c_3}{(.7,.8,.0)}, \frac{c_4}{(.0,.0,.0)}\}, \{\frac{h_1}{(.0,.0,.0)}, \frac{h_2}{(1,.1,.0)}, \frac{h_3}{(.9,.2,.5)}\})),$$
$$(x_2, (\{\frac{u_1}{(.1,.5,.3)}, \frac{u_2}{(.1,.8,.9)}, \frac{u_3}{(.0,.0,1)}, \frac{u_4}{(.2,.8,.5)}\}, \{\frac{c_1}{(.5,.5,5)}, \frac{c_2}{(.5,.3,.7)}, \frac{c_3}{(.5,.4,.3)}, \frac{c_4}{(.1,1,1)}\}, \{\frac{h_1}{(1,.2,.5)}, \frac{h_2}{(1,1,1)}, \frac{h_3}{(.1,.8,.6)}\})),$$

$$N_B = \{(x_1, (\{\frac{u_1}{(.3,.7,.2)}, \frac{u_2}{(.4,.3,.8)}, \frac{u_3}{(.6,.5,.4)}, \frac{u_4}{(.6,.7,.4)}\}, \{\frac{c_1}{(.5,.6,.8)}, \frac{c_2}{(.5,.7,.8)}, \frac{c_3}{(.3,.5,.6)}, \frac{c_4}{(1,.0,.0)}\}, \{\frac{h_1}{(1,.0,.1)}, \frac{h_2}{(.5,.6,.3)}, \frac{h_3}{(1,.0,.0)}\})),$$
$$(x_2, (\{\frac{u_1}{(.7,.3,.5)}, \frac{u_2}{(.6,.7,.8)}, \frac{u_3}{(.6,.8,.6)}, \frac{u_4}{(.6,.7,.3)}\}, \{\frac{c_1}{(.4,.3,.2)}, \frac{c_2}{(.5,.6,.7)}, \frac{c_3}{(.9,.1,.3)}, \frac{c_4}{(1,.2,.1)}\}, \{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.0,..1)}, \frac{h_3}{(.4,.2,.3)}\})),$$
$$(x_3, (\{\frac{u_1}{(.6,.3,.6)}, \frac{u_2}{(.3,.2,.6)}, \frac{u_3}{(.6,.7,.5)}, \frac{u_4}{(.3,.7,.6)}\}, \{\frac{c_1}{(.7,.5,.3)}, \frac{c_2}{(.6,.7,.2)}, \frac{c_3}{(.5,.4,.5)}, \frac{c_4}{(.3,.6,.5)}\}, \{\frac{h_1}{(.3,.5,.6)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.3,.2,.7)}\}))\}$$

$$N_A \sqcap N_B =$$
$$(x_1, (\{\frac{u_1}{(.3,.7,.4)}, \frac{u_2}{(.2,.4,.8)}, \frac{u_3}{(.3,.5,.5)}, \frac{u_4}{(.6,.8,.4)}\}, \{\frac{c_1}{(.5,.6,.8)}, \frac{c_2}{(.2,.7,.8)}, \frac{c_3}{(.3,.8,.6)}, \frac{c_4}{(1,.0,.0)}\}, \{\frac{h_1}{(.0,.0,.1)}, \frac{h_2}{(.5,1,.3)}, \frac{h_3}{(.9,.2,.5)}\})),$$
$$(x_2, (\{\frac{u_1}{(.1,.5,.5)}, \frac{u_2}{(.1,.8,.9)}, \frac{u_3}{(.0,.8,.6)}, \frac{u_4}{(.2,.7,.5)}\}, \{\frac{c_1}{(.4,.5,.5)}, \frac{c_2}{(.5,.6,.7)}, \frac{c_3}{(.5,.4,.3)}, \frac{c_4}{(.1,1,1)}\}, \{\frac{h_1}{(1,.2,.5)}, \frac{h_2}{(1,1,1)}, \frac{h_3}{(.1,.8,.6)}\})),$$

Proposition 3.15. Let N_A, N_B and N_C are three Nsm-sets. Then

i. $N_A \sqcap (N_B \sqcap N_C) = (N_A \sqcap N_B) \sqcap N_C$
ii. $N_A \sqcap N_A = N_A$
iii. $N_A \sqcap N_{A\emptyset} = N_A$
iv. $N_A \sqcap N_{B\emptyset} = N_A$

Proof: The proof is straightforward.

4. NS-multi-set Decision Making

In this section we recall the algorithm designed for solving a neutrosophic soft set and based on algorithm proposed by Alkazaleh and Saleh [20] for solving fuzzy soft multisets based decision making problem, we propose a new algorithm to solve neutrosophic soft multiset(NS-mset) based decision-making problem.
Now the algorithm for most appropriate selection of an object will be as follows.

4-1 Algorithm (Maji's algorithm using scores)

Maji [20] used the following algorithm to solve a decision-making problem.

(1) input the neutrosophic Soft Set (F, A).
(2) input P, the choice parameters of Mrs. X which is a subset of A.
(3) consider the NSS (F, P) and write it in tabular form.
(4) compute the comparison matrix of the NSS (F, P).
(5) compute the score S_i, for all i using $S_i = T_i + I_i - F_i$
(6) find $S_k = \max x_i \; S_i$
(7) if k has more than one value then any one of bi may be chosen.

4.2 NS-multiset Theoretic Approch to Decision–Making Problem

In this section, we construct a Ns-mutiset decision making method by the following algorithm;

(1) Input the neutrosophic soft multiset (H, C) which is introduced by making any operations between (F, A) and (G, B).
(2) Apply MA to the first neutrosophic soft multi-set part in (H, C) to get the decision S_{k_1}.
(3) Redefine the neutrosophic soft multiset (H, C) by keeping all values in each row where S_{k_1} is maximum and replacing the values in the other rows by zero, to get $(H, C)_1$.
(4) Apply MA to the second neutrosophic soft multiset part in$(H, C)_1$to get the decision S_{k_2}.
(5) Redefine the neutrosophic soft set$(H, C)_1$ by keeping the first and second parts and apply the method in step (c) to the third part.
(6) Apply MA to the third neutrosophic soft multiset part in $(H, C)_2$ to get the decision S_{k_3}.
(7) The decision is $(S_{k_1}, S_{k_2}, S_{k_3})$.

5-Application in a Decision Making Problem

Assume that U_1= $\{u_1,u_2,u_3,u_4\}$, U_2= $\{c_1,c_2,c_3,c_4\}$ and U_3 ={ h_1 , h_2,h_3 } be the sets of es" ,"cars", and "hotels" , respectively and $\{E_1, E_2, E_3\}$be a collection of sets of decision parameters related to the above universe, where

E_1= $\{x_1{}^{U_1}$ = expensive, $x_2{}^{U_1}$ = cheap, $x_3{}^{U_1}$ = wooden},

E_2=
$\{x_1{}^{U_2}$ = expensive, $x_2{}^{U_2}$ =
in green surroundings, $x_3{}^{U_2}$ = sporty}
and

E_3= $\{x_1{}^{U_3}$=expensive, $x_2{}^{U_3}$ = majestic, $x_3{}^{U_3}$ =
in Kuala Lumpur}

Let A=$\{x_1 = \{x_1{}^{U_1}, x_1{}^{U_2}, x_1{}^{U_3}\}, x_2 = \{x_2{}^{U_1}, x_2{}^{U_2}, x_2{}^{U_3}\}, x_4 = \{x_3{}^{U_1}, x_2{}^{U_2}, x_1{}^{U_3}\}\}$

and
B=$\{x_1 = \{x_1{}^{U_1}, x_1{}^{U_2}, x_1{}^{U_3}\}, x_2 = \{x_2{}^{U_1}, x_2{}^{U_2}, x_2{}^{U_3}\}, x_3 = \{x_3{}^{U_1}, x_3{}^{U_2}, x_3{}^{U_3}\}\}$

Suppose that a person wants to choose objects from the set of given objects with respect to the sets of choices parameters. Let there be two observation N_A and N_B by two expert Y_1 and Y_2, respectively.

$$N_A=\{(x_1,(\{\tfrac{u}{(.5,.3,.4)},\tfrac{u_2}{(.2,.4,.4)},\tfrac{u_3}{(.3,.3,.5)},\tfrac{u_4}{(.7,.8,.4)}\},\{\tfrac{c}{(.7,.1,.5)},\tfrac{c_2}{(.2,.5,.7)},\tfrac{c_3}{(.7,.8,.0)},\tfrac{c_4}{(.0,.0,.0)}\},\{\tfrac{h}{(.0,.0,.0)},\tfrac{h_2}{(1,.1,.0)},\tfrac{h_3}{(.9,.2,.5)}\})),(x_2,(\{\tfrac{u}{(.1,.5,.3)},\tfrac{u_2}{(.1,.8,.9)},\tfrac{u_3}{(.0,.1,1)},\tfrac{u_4}{(.2,.8,.5)}\},\{\tfrac{c}{(.5,.5,5)},\tfrac{c_2}{(.5,.3,.7)},\tfrac{c_3}{(.5,.4,.3)},\tfrac{c_4}{(.1,1,1)}\},\{\tfrac{h}{(1,.2,.5)},\tfrac{h_2}{(1,1,1)},\tfrac{h_3}{(.1,.8,.6)}\})),(x_4,(\{\tfrac{u}{(.2,.5,.6)},\tfrac{u_2}{(.6,.2,.3)},\tfrac{u_3}{(.8,.7,.6)},\tfrac{u_4}{(.3,.7,.6)}\},\{\tfrac{c}{(.3,.5,.7)},\tfrac{c_2}{(.3,.6,.2)},\tfrac{c_3}{(.8,.5,.3)},\tfrac{c_4}{(.3,.5,.5)}\},\{\tfrac{h}{(.2,.6,.5)},\tfrac{h_2}{(1,.0,.0)},\tfrac{h_3}{(.5,.2,.3)}\})\}$$

$$N_B=\{(x_1,(\{\tfrac{u}{(.3,.7,.2)},\tfrac{u_2}{(.4,.3,.8)},\tfrac{u_3}{(.6,.5,.4)},\tfrac{u_4}{(.6,.7,.4)}\},\{\tfrac{c}{(.5,.6,.8)},\tfrac{c_2}{(.5,.7,.8)},\tfrac{c_3}{(.3,.5,.6)},\tfrac{c_4}{(1,.0,.0)}\},\{\tfrac{h}{(1,.0,.1)},\tfrac{h_2}{(.5,.6,.3)},\tfrac{h_3}{(1,.0,.0)}\})),(x_2,(\{\tfrac{u}{(.7,.3,.5)},\tfrac{u_2}{(.6,.7,.8)},\tfrac{u_3}{(.6,.8,.6)},\tfrac{u_4}{(.6,.7,.3)}\},\{\tfrac{c}{(.4,.3,.2)},\tfrac{c_2}{(.5,.6,.7)},\tfrac{c_3}{(.9,.1,.3)},\tfrac{c_4}{(1,.2,.1)}\},\{\tfrac{h}{(1,.0,.0)},\tfrac{h_2}{(1,.0,.1)},\tfrac{h_3}{(.4,.2,.3)}\})),(x_3,(\{\tfrac{u}{(.6,.3,.6)},\tfrac{u_2}{(.3,.2,.6)},\tfrac{u_3}{(.6,.7,.5)},\tfrac{u_4}{(.3,.7,.6)}\},\{\tfrac{c}{(.7,.5,.3)},\tfrac{c_2}{(.6,.7,.2)},\tfrac{c_3}{(.5,.4,.5)},\tfrac{c_4}{(.3,.6,.5)}\},\{\tfrac{h}{(.3,.5,.6)},\tfrac{h_2}{(1,.0,.0)},\tfrac{h_3}{(.3,.2,.7)}\})\}$$

$$N_A \sqcup N_B=\{(x_1,(\{\tfrac{u_1}{(.5,.3,.4)},\tfrac{u_2}{(.4,.3,.4)},\tfrac{u_3}{(.6,.3,.4)},\tfrac{u_4}{(.7,.7,.4)}\},\{\tfrac{c_1}{(.7,.1,.5)},\tfrac{c_2}{(.5,.5,.7)},\tfrac{c_3}{(.7,.3,.0)},\tfrac{c_4}{(1,.0,.0)}\},\{\tfrac{h_1}{(1,.0,.0)},\tfrac{h_2}{(1,.1,.0)},\tfrac{h_3}{(.9,.2,.5)}\})),(x_2,(\{\tfrac{u_1}{(.7,.3,.5)},\tfrac{u_2}{(.6,.7,.8)},\tfrac{u_3}{(.6,.0,.6)},\tfrac{u_4}{(.6,.7,.3)}\},\{\tfrac{c_1}{(.5,.3,.2)},\tfrac{c_2}{(.5,.3,.7)},\tfrac{c_3}{(.9,.1,.3)},\tfrac{c_4}{(1,.2,.1)}\},\{\tfrac{h_1}{(1,.0,.0)},\tfrac{h_2}{(1,.0,.1)},\tfrac{h_3}{(.4,.2,.3)}\})$$

$(x_3, (\{\frac{u_1}{(.6,.3,.6)}, \frac{u_2}{(.3,.2,.6)}, \frac{u_3}{(.6,.7,.5)}, \frac{u_4}{(.3,.7,.6)}\},$
$\{\frac{c_1}{(.7,.5,.3)}, \frac{c_2}{(.6,.7,.2)}, \frac{c_3}{(.5,.4,.5)}, \frac{c_4}{(.3,.6,.5)}\},$
$\{\frac{h_1}{(.3,.5,.6)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.3,.2,.7)}\})),$
$(x_4, (\{\frac{u_1}{(.2,.5,.6)}, \frac{u_2}{(.6,.2,.3)}, \frac{u_3}{(.8,.7,.6)}, \frac{u_4}{(.3,.7,.6)}\},$
$\{\frac{c_1}{(.3,.5,.7)}, \frac{c_2}{(.3,.6,.2)}, \frac{c_3}{(.8,.5,.3)}, \frac{c_4}{(.3,.5,.5)}\},$
$\{\frac{h_1}{(.2,.6,.5)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.5,.2,.3)}\}))\}$

Now we apply MA to the first neutrosophic soft multiset part in (H,D) to take the decision from the availability set U_1. The tabular representation of the first resultant neutrosophic soft multiset part will be as in Table 1. The comparison table for the first resultant neutrosophic soft multiset part will be as in Table 2.

Next we compute the row-sum, column-sum, and the score for each u_i as shown in Table 3.

From Table 3, it is clear that the maximum score is 6, scored by u_3.

Table 1 :Tabular representation: U_1- neutrosophic soft multiset part of (H, D).

U_1	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
u_1	(.5 ,.3 ,.4)	(.7 ,.3 ,.5)	(.6 ,.3 ,.6)	(.2 ,.5 ,.6)
u_2	(.4 ,.3 ,.4)	(.6 ,.7 ,.8)	(.3 ,.2 ,.6)	(.6 ,.2 ,.3)
u_3	(.6 ,.3 ,.4)	(.6 ,.0 ,.6)	(.6 ,.7 ,.5)	(.8 ,.7 ,.6)
u_4	(.7 ,.7 ,.4)	(.6 ,.7 ,.3)	(.3 ,.7 ,.6)	(.3 ,.7 ,.6)

Table 2 :Comparison table: U_1- neutrosophic soft multiset part of (H, D).

U_1	u_1	u_2	u_3	u_4
u_1	4	3	1	1
u_2	2	4	1	1
u_3	3	3	4	3
u_4	3	2	1	4

Table 3 :Score table: U_1- neutrosophic soft multiset part of (H, D).

	Row sum	Column sum	Score
u_1	9	12	-3
u_2	8	12	-4
u_3	13	7	**6**
u_4	10	9	1

Now we redefine the neutrosophic soft multiset (H, D) by keeping all values in each row where u_3 is maximum and replacing the values in the other rows by zero (1, 0 , 0):

$(H, D)_1 = \{(x_1, (\{\frac{u_1}{(.5,.3,.4)}, \frac{u_2}{(.4,.3,.4)}, \frac{u_3}{(.6,.3,.4)}, \frac{u_4}{(.7,.7,.4)}\},$
$\{\frac{c_1}{(.7,.1,.5)}, \frac{c_2}{(.5,.5,.7)}, \frac{c_3}{(.7,.3,.0)}, \frac{c_4}{(1,.0,.0)}\},$
$\{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.1,.0)}, \frac{h_3}{(.9,.2,.5)}\})),$
$(x_2, (\{\frac{u_1}{(.7,.3,.5)}, \frac{u_2}{(.6,.7,.8)}, \frac{u_3}{(.6,.0,.6)}, \frac{u_4}{(.6,.7,.3)}\},$
$\{\frac{c_1}{(1,.0,.0)}, \frac{c_2}{(1,.0,.0)}, \frac{c_3}{(1,.0,.0)}, \frac{c_4}{(1,.0,.0)}\},$
$\{\frac{h_1}{(1,.0,.0)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(1,.0,.0)}\})),$
$(x_3, (\{\frac{u_1}{(.6,.3,.6)}, \frac{u_2}{(.3,.2,.6)}, \frac{u_3}{(.6,.7,.5)}, \frac{u_4}{(.3,.7,.6)}\},$
$\{\frac{c_1}{(.7,.5,.3)}, \frac{c_2}{(.6,.7,.2)}, \frac{c_3}{(.5,.4,.5)}, \frac{c_4}{(.3,.6,.5)}\},$
$\{\frac{h_1}{(.3,.5,.6)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.3,.2,.7)}\})),$
$(x_4, (\{\frac{u_1}{(.2,.5,.6)}, \frac{u_2}{(.6,.2,.3)}, \frac{u_3}{(.8,.7,.6)}, \frac{u_4}{(.3,.7,.6)}\},$
$\{\frac{c_1}{(.3,.5,.7)}, \frac{c_2}{(.3,.6,.2)}, \frac{c_3}{(.8,.5,.3)}, \frac{c_4}{(.3,.5,.5)}\},$
$\{\frac{h_1}{(.2,.6,.5)}, \frac{h_2}{(1,.0,.0)}, \frac{h_3}{(.5,.2,.3)}\}))\}$

U_2	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
c_1	(.7 ,.1 ,.5)	(1 ,.0 ,.0)	(.7 ,.5 ,.3)	(.3 ,.5 ,.67)
c_2	(.5 ,.5 ,.7)	(1 ,.0 ,.0)	(.6 ,.7 ,.2)	(.3 ,.6 ,.2)
c_3	(.7 ,.3 ,.0)	(1 ,.0 ,.0)	(.5 ,.4 ,.5)	(.8 ,.5 ,.3)
c_4	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(.3 ,.6 ,.5)	(.3 ,.5 ,.5)

Table 4 :Tabular representation: U_2- neutrosophic soft multiset part of $(H, D)_1$.

Table 5 :Comparison table: U_2- neutrosophic soft multiset part of $(H, D)_1$

U_2	c_1	c_2	c_3	c_4
c_1	4	2	2	2
c_2	4	4	3	3
c_3	3	3	4	4
c_4	2	2	3	4

Table 6 :Score table: U_2- neutrosophic soft multiset part of $(H, D)_1$

	Row sum	Column sum	Score
c_1	10	13	-3
c_2	14	11	**3**
c_3	14	12	2
c_4	11	13	-2

Now we apply MA to the second neutrosophic soft multiset part in $(H, D)_1$ to take the decision from the availability set U_2. The tabular representation of the first resultant neutrosophic soft multiset part will be as in Table 4.

The comparison table for the first resultant neutrosophic soft multiset part will be as in

Table 5.

Next we compute the row-sum, column-sum, and the score for each u_i as shown in Table 3.

From Table 6, it is clear that the maximum score is 3, scored by c_2.

Now we redefine the neutrosophic soft multiset $(H, D)_2$by keeping all values in each row where c_2 is maximum and replacing the values in the other rows by zero (1, 0 , 0):

$$(H, D)_2 = \{(x_1, (\{\tfrac{u_1}{(.5,.3,.4)}, \tfrac{u_2}{(.4,.3,.4)}, \tfrac{u_3}{(.6,.3,.4)}, \tfrac{u_4}{(.7,.7,.4)}\}, \{\tfrac{c_1}{(.7,.1,.5)}, \tfrac{c_2}{(.5,.5,.7)}, \tfrac{c_3}{(.7,.3,.0)}, \tfrac{c_4}{(1,.0,.0)}\}, \{\tfrac{h_1}{(1,.0,.0)}, \tfrac{h_2}{(1,.1,.0)}, \tfrac{h_3}{(.9,.2,.5)}\})),$$
$$(x_2, (\{\tfrac{u_1}{(.7,.3,.5)}, \tfrac{u_2}{(.6,.7,.8)}, \tfrac{u_3}{(.6,.0,.6)}, \tfrac{u_4}{(.6,.7,.3)}\}, \{\tfrac{c_1}{(1,.0,.0)}, \tfrac{c_2}{(1,.0,.0)}, \tfrac{c_3}{(1,.0,.0)}, \tfrac{c_4}{(1,.0,.0)}\}, \{\tfrac{h_1}{(1,.0,.0)}, \tfrac{h_2}{(1,.0,.0)}, \tfrac{h_3}{(1,.0,.0)}\}))$$
$$(x_3, (\{\tfrac{u_1}{(.6,.3,.6)}, \tfrac{u_2}{(.3,.2,.6)}, \tfrac{u_3}{(.6,.7,.5)}, \tfrac{u_4}{(.3,.7,.6)}\}, \{\tfrac{c_1}{(.7,.5,.3)}, \tfrac{c_2}{(.6,.7,.2)}, \tfrac{c_3}{(.5,.4,.5)}, \tfrac{c_4}{(.3,.6,.5)}\}, \{\tfrac{h_1}{(.3,.5,.6)}, \tfrac{h_2}{(1,.0,.0)}, \tfrac{h_3}{(.3,.2,.7)}\})),$$
$$(x_4, (\{\tfrac{u_1}{(.2,.5,.6)}, \tfrac{u_2}{(.6,.2,.3)}, \tfrac{u_3}{(.8,.7,.6)}, \tfrac{u_4}{(.3,.7,.6)}\}, \{\tfrac{c_1}{(.3,.5,.7)}, \tfrac{c_2}{(.3,.6,.2)}, \tfrac{c_3}{(.8,.5,.3)}, \tfrac{c_4}{(.3,.5,.5)}\}, \{\tfrac{h_1}{(1,.0,.0)}, \tfrac{h_2}{(1,.0,.0)}, \tfrac{h_3}{(1,.0,.0)}\}))\}$$

U_3	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$d_{1,4}$
h_1	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(.3 ,.5 ,.6)	(1 ,.0 ,.0)
h_2	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(1 ,.0 ,.0)
h_3	(1 ,.0 ,.0)	(1 ,.0 ,.0)	(.3 ,.2 ,.7)	(1 ,.0 ,.0)

Table 7: Tabular representation: U_3- neutrosophic soft multiset part of $(H, D)_2$.

Table 8 :Comparison table: U_3- neutrosophic soft multiset part of $(H, D)_2$

U_3	h_1	h_2	h_3
h_1	3	3	4
h_2	4	3	4
h_3	3	3	3

Table 9 :Score table: U_3- neutrosophic soft multiset part of $(H, D)_2$

	Row sum	Column sum	Score
h_1	10	10	0
h_2	11	9	**2**
h_3	9	11	-2

Now we apply MA to the third neutrosophic soft multiset part in $(H, D)_2$ to take the decision from the availability set U_3. The tabular representation of the first resultant neutrosophic soft multiset part will be as in Table 7. The comparison table for the first resultant neutrosophic soft multiset part will be as in Table 8. Next we compute the row-sum, column-sum, and the score for each u_i as shown in Table 3. From Table 9, it is clear that the maximum score is 2, scored by h_2. Then from the above results the decision for Mr.X is **(u_3 , c_2, h_2).**

6. Conclusion

In this work, we present neutrosophic soft multi-set theory and study their properties and operations. Then, we give a decision making methods. An application of this method in decsion making problem is shown.

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Abstract

This book is a collection of nine papers, contributed by different authors and co-authors (listed in the order of the papers): A. A. Salama, O. M. Khaled, K. M. Mahfouz, M. Ali, F. Smarandache, M. Shabir, L. Vladareanu, S. Broumi, K. Mondal, S. Pramanik, I. Arockiarani, I. R. Sumathi, M. Eisa and I. Deli.

In first paper, the authors studied Neutrosophic Correlation and Simple Linear Regression. The Generalization of Neutrosophic Rings and Neutrosophic Fields are proposed in the second paper. Cosine Similarity Measure of Interval Valued Neutrosophic Sets are studied in third paper. In fourth paper A Study on Problems of Hijras in West Bengal Based on Neutrosophic Cognitive Maps are introduced. Similarly in fifth paper Neutrosophic Crisp Set Theory is discussed. In paper six Interval Valued Fuzzy Neutrosophic Soft Structure Spaces are presented by the authors. Soft Neutrosophic Bi-LA-Semigroup and Soft Neutrosophic N-LA-Semigroup is given in seventh paper. Introduction to Image Processing via Neutrosophic Technique is given in paper eight. In the last paper, Neutrosophic Soft Multi-Set Theory and Its Decision Making is presented by the authors.

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